Prb. #	ANSWERS

1

(a) $Q = C\Delta V = (4.00\times10^{-6}\text{ F})(12.0\text{ V}) = 4.80\times10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = (4.00\times10^{-6}\text{ F})(1.50\text{ V}) = 6.00\times10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

Note: Throughout this assignment, the given values of certain parameters, such as C and V, may vary

2

(a) $C = \dfrac{\kappa\,\epsilon_0\,A}{d} = \dfrac{(1.00)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.30\times10^{-4}\text{ m}^2)}{1.50\times10^{-3}\text{ m}}$

$= 1.36\times10^{-12}\text{ F} = \boxed{1.36\text{ pF}}$

(b) $Q = C\Delta V = (1.36\text{ pF})(12.0\text{ V}) = \boxed{16.3\text{ pC}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{12.0\text{ V}}{1.50\times10^{-3}\text{ m}} = \boxed{8.00\times10^{3}\text{ V/m}}$

3

(a) For a spherical capacitor with inner radius a and outer radius b,

$$C = \frac{ab}{k_e(b-a)} = \frac{(0.0700\text{ m})(0.140\text{ m})}{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(0.140\text{ m} - 0.0700\text{ m})}$$

$= \boxed{15.6\text{ pF}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{1.56\times10^{-11}\text{ F}} = 2.57\times10^{5}\text{ V} = \boxed{257\text{ kV}}$

(c) Solution to **What If?**

Set the capacitance of a cylindrical capacitor, given by $C = \dfrac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}$, equal to that of a spherical capacitor, given by $C = \dfrac{ab}{k_e(b-a)}$, and solve for the length of the cylindrical capacitor.

$$C_{\text{cylinder}} = C_{\text{sphere}} \rightarrow \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)} = \frac{ab}{k_e(b-a)} \rightarrow \ell = \frac{2ab}{(b-a)}\ln\left(\frac{b}{a}\right)$$

Substituting numerical values,

$$\ell = \frac{2(6.70\text{ cm})(15.0\text{ cm})}{(15.0\text{ cm} - 6.70\text{ cm})}\ln\left(\frac{15.0\text{ cm}}{6.70\text{ cm}}\right) = 19.5\text{ cm}$$

4

(a) When connected in series, the equivalent capacitance is

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$, or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.20\ \mu\text{F}} + \frac{1}{8.50\ \mu\text{F}} \rightarrow C_{eq} = \boxed{2.81\ \mu\text{F}}$$

(b) When connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 = 4.20\ \mu\text{F} + 8.50\ \mu\text{F} = \boxed{12.70\ \mu\text{F}}$$

5

(a) $C = \dfrac{\kappa\epsilon_0 A}{d} = \dfrac{2.10(8.85\times10^{-12}\text{ F/m})(1.75\times10^{-4}\text{ m}^2)}{4.00\times10^{-5}\text{ m}} = 8.13\times10^{-11}\text{F}$

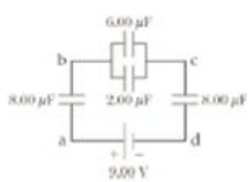
$= \boxed{81.3\text{ pF}}$

(b) $\Delta V_{max} = E_{max}d = (60.0\times10^{6}\text{ V/m})(4.00\times10^{-5}\text{ m}) = \boxed{2.40\text{ kV}}$

6

(a) First, we replace the parallel combination between points b and c by its equivalent capacitance,
$C_{bc} = 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = 8.00\ \mu\text{F}$.
Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore

$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu\text{F}}$$



ANS. FIG. P26.26

giving

$$C_{eq} = \frac{8.00\ \mu\text{F}}{3} = \boxed{2.67\ \mu\text{F}}$$

(b) The charge on each capacitor in the series is the same as the charge on the equivalent capacitor:

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu\text{F})(9.00\text{ V}) = 24.0\ \mu\text{C}$$

Then, note that $\Delta V_{bc} = \dfrac{Q_{bc}}{C_{bc}} = \dfrac{24.0\ \mu\text{C}}{8.00\ \mu\text{F}} = 3.00\text{ V}$. The charge on each capacitor in the original circuit is:

On the 8.00 μF between a and b:

$Q_8 = Q_{ab} = \boxed{24.0\ \mu\text{C}}$

On the 8.00 μF between c and d:

$Q_8 = Q_{cd} = \boxed{24.0\ \mu\text{C}}$

On the 2.00 μF between b and c:

$Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu\text{F})(3.00\text{ V}) = \boxed{6.00\ \mu\text{C}}$

On the 6.00 μF between b and c:

$Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu\text{F})(3.00\text{ V}) = \boxed{18.0\ \mu\text{C}}$

(c) We earlier found that $\Delta V_{bc} = 3.00\text{ V}$. The two 8.00 μF capacitors have the same voltage: $\Delta V_8 = \Delta V_8 = \dfrac{Q}{C} = \dfrac{24.0\ \mu\text{C}}{8.00\ \mu\text{F}} = 3.00\text{ V}$, so we conclude that the potential difference across each capacitor is the same: $\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\text{ V}}$.

7

(a) We simplify the circuit of Figure P26.23 in three steps as shown in ANS. FIG. P26.23 panels (a), (b), and (c). First, the 15.0-μF and 3.00-μF capacitors in series are equivalent to

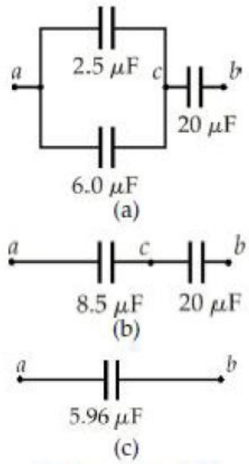
$$\frac{1}{(1/15.0\ \mu\text{F})+(1/3.00\ \mu\text{F})}=2.50\ \mu\text{F}$$

Next, the 2.50-μF capacitor combines in parallel with the 6.00-μF capacitor, creating an equivalent capacitance of 8.50 μF. At last, this 8.50-μF equivalent capacitor and the 20.0-μF capacitor are in series, equivalent to

$$\frac{1}{(1/8.50\ \mu\text{F})+(1/20.00\ \mu\text{F})}=\boxed{5.96\ \mu\text{F}}$$

(b) We find the charge on each capacitor and the voltage across each by working backwards through solution figures (c)–(a), alternately applying $Q=C\Delta V$ and $\Delta V=Q/C$ to every capacitor, real or equivalent. For the 5.96-μF capacitor, we have

$$Q=C\Delta V=(5.96\ \mu\text{F})(15.0\ \text{V})=\boxed{89.5\ \mu\text{C}}$$



ANS. FIG. P26.23

Thus, if a is higher in potential than b, just 89.5 μC flows between the wires and the plates to charge the capacitors in each picture. In (b) we have, for the 8.5-μF capacitor,

$$\Delta V_{ac}=\frac{Q}{C}=\frac{89.5\ \mu\text{C}}{8.50\ \mu\text{F}}=10.5\ \text{V}$$

and for the 20.0-μF capacitor in (b), (a), and the original circuit, we have $Q_{20}=89.5\ \mu$C. Then

$$\Delta V_{cb}=\frac{Q}{C}=\frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}}=4.47\ \text{V}$$

Next, the circuit in diagram (a) is equivalent to that in (b), so $\Delta V_{cb}=4.47$ V and $\Delta V_{ac}=10.5$ V.

For the 2.50-μF capacitor, $\Delta V=10.5$ V and

$$Q=C\Delta V=(2.50\ \mu\text{F})(10.5\ \text{V})=\boxed{26.3\ \mu\text{C}}$$

For the 6.00-μF capacitor, $\Delta V=10.5$ V and

$$Q_6=C\Delta V=(6.00\ \mu\text{F})(10.5\ \text{V})=\boxed{63.2\ \mu\text{C}}$$

Now, 26.3 μC having flowed in the upper parallel branch in (a), back in the original circuit we have $Q_{15}=26.3\ \mu$C and $Q_3=26.3\ \mu$C.

8

(a) $U_E=\frac{1}{2}C(\Delta V)^2=\frac{1}{2}(3.00\ \mu\text{F})(12.0\ \text{V})^2=\boxed{216\ \mu\text{J}}$

(b) $U_E=\frac{1}{2}C(\Delta V)^2=\frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2=\boxed{54.0\ \mu\text{J}}$

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From $U_E=\frac{1}{2}C\Delta V^2$, we have

$$U_E=\frac{1}{2}(31.8\times10^{-6}\ \text{F})(4.35\times10^{3}\ \text{V})^2=301\ \text{J}$$

10

(a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{18.0\ \mu\text{F}}+\frac{1}{36.0\ \mu\text{F}}\rightarrow C_{eq}=\boxed{12.0\ \mu\text{F}}$$

(b) This series combination is connected to a 12.0-V battery, the total stored energy is

$$U_{E,\,eq}=\frac{1}{2}C_{eq}(\Delta V)^2=\frac{1}{2}(12.0\times10^{-6}\ \text{F})(12.0\ \text{V})^2=\boxed{8.64\times10^{-4}\ \text{J}}$$

(c) Capacitors in series carry the same charge as their equivalent capacitor. The charge stored on each of the two capacitors in the series combination is

$$Q_1=Q_2=Q_{total}=C_{eq}(\Delta V)=(12.0\ \mu\text{F})(12.0\ \text{V})$$
$$=144\ \mu\text{C}=1.44\times10^{-4}\ \text{C}$$

and the energy stored in each of the individual capacitors is:

18.0 μF capacitor:

$$U_{E1}=\frac{Q_1^2}{2C_1}=\frac{(1.44\times10^{-4}\ \text{C})^2}{2(18.0\times10^{-6}\ \text{F})}=\boxed{5.76\times10^{-4}\ \text{J}}$$

36.0 μF capacitor:

$$U_{E2}=\frac{Q_2^2}{2C_2}=\frac{(1.44\times10^{-4}\ \text{C})^2}{2(36.0\times10^{-6}\ \text{F})}=\boxed{2.88\times10^{-4}\ \text{J}}$$

(d) $U_{E1}+U_{E2}=5.76\times10^{-4}\ \text{J}+2.88\times10^{-4}\ \text{J}=8.64\times10^{-4}\ \text{J}=U_{E,\,eq}$, which is one reason why the 12.0 μF capacitor is considered to be equivalent to the two capacitors.

(e) The total energy of the equivalent capacitance will always equal the sum of the energies stored in the individual capacitors.

(f) If C_1 and C_2 were connected in parallel rather than in series, the equivalent capacitance would be $C_{eq}=C_1+C_2=18.0\ \mu\text{F}+36.0\ \mu\text{F}=54.0\ \mu\text{F}$. If the total energy stored in this parallel combination is to be the same as stored in the original series combination, it is necessary that

$$\frac{1}{2}C_{eq}(\Delta V)^2=U_{E,\,eq}$$

From which we obtain

$$\Delta V=\sqrt{\frac{2U_{E,\,eq}}{C_{eq}}}=\sqrt{\frac{2(8.64\times10^{-4}\ \text{J})}{54.0\times10^{-6}\ \text{F}}}=\boxed{5.66\ \text{V}}$$

(g) Because the potential difference is the same across the two capacitors when connected in parallel, and $U_E=\frac{1}{2}C(\Delta V)^2$, the larger capacitor C_2 stores more energy.