## ANSWERS

Note: Given values in this assignment may vary.

**1**

From $|\mathcal{E}| = L\left(\dfrac{\Delta i}{\Delta t}\right)$, we have

$$L = \frac{|\mathcal{E}|}{|\Delta i/\Delta t|} = \frac{|\mathcal{E}|(\Delta t)}{|\Delta i|} = \frac{(12.0\times10^{-3}\text{ V})(0.500\text{ s})}{|2.00\text{ A} - 3.50\text{ A}|}$$
$$= 4.00\times10^{-3}\text{ H} = \boxed{4.00\text{ mH}}$$

**2**

The inductance is $L = \dfrac{\mu_0 N^2 A}{\ell}$ with $A = \pi r^2$. The induced emf as a function of time is $\mathcal{E}_L = -L\dfrac{di}{dt}$. By substitution we have

$$\mathcal{E}_L = -L\frac{di}{dt} = -\frac{\mu_0 N^2 \pi r^2}{\ell}\frac{di}{dt} \quad\text{and}\quad r = \left(\frac{-\mathcal{E}_L \ell}{\mu_0 N^2 \pi\, di/dt}\right)^{1/2}$$

Then $$r = \left(\frac{-(175\times10^{-6}\text{ V})(0.160\text{ m})}{(4\pi\times10^{-7}\text{ N/A}^2)(420)^2\pi(-0.421\text{ A/s})}\right)^{1/2} = \boxed{9.77\text{ mm}}$$

**3**

(a) The inductance of a solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 N^2 \pi r^2}{\ell} = \frac{\mu_0 (510)^2 \pi \left(8.00\times10^{-3}\text{ m}\right)^2}{0.140\text{ m}}$$
$$= 4.69\times10^{-4}\text{ H} = \boxed{0.469\text{ mH}}$$

(b) The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{4.69\times10^{-4}\text{ H}}{2.50\ \Omega} = 1.88\times10^{-4}\text{ s} = \boxed{0.188\text{ ms}}$$

**4**

At time t:

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) \quad \text{where } \tau = \frac{2.7}{20} = 0.135$$

After a long time,

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\infty}\right) = \frac{\varepsilon}{R}$$

a) Current reach 50% of its final value: $i(t) = \dfrac{50}{100} i$ ,

Thus $\dfrac{i(t)}{i} = 0.5$ :

$$\frac{i(t)}{i} = \frac{\varepsilon/_R\,(1 - e^{-\frac{t}{\tau}})}{\varepsilon/_R} = (1 - e^{-\frac{t}{\tau}})$$

Solve $0.5 = (1 - e^{-\frac{t}{0.135}})$ , note: $\tau = \frac{2.7}{20} = 0.135$

$$e^{-\frac{t}{0.135}} = 1 - 0.5 = 0.5 \rightarrow$$
$$\ln(0.5) = -\frac{t}{0.135} \rightarrow 0.69 = \frac{t}{0.135} \rightarrow \boxed{t = 0.09\text{ (s)}}$$

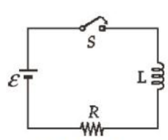
b) Current reach 90% of its final value: $i(t) = \dfrac{90}{100} i$ ,

Thus $\dfrac{i(t)}{i} = 0.9$

Similarly solve $0.9 = (1 - e^{-\frac{t}{0.135}})$

$$e^{-\frac{t}{0.135}} = 1 - 0.9 = 0.1 \rightarrow$$
$$\ln(0.1) = -\frac{t}{0.135} \rightarrow 2.3 = \frac{t}{0.135} \rightarrow$$
$$\boxed{t = 0.31\text{ (s)}}$$

**5**




(a) $\tau = \dfrac{L}{R} = \dfrac{8.00\times10^{-3}\text{ H}}{4.00\ \Omega} = 2.00\times10^{-3}\text{ s} = \boxed{2.00\text{ ms}}$

(b) $i = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\text{ A}}$

(c) $I_i = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{4.00\ \Omega} = \boxed{1.50\text{ A}}$

(d) $0.800 = 1 - e^{-t/2.00\text{ ms}} \rightarrow t = -(2.00\text{ ms})\ln(0.200) = \boxed{3.22\text{ ms}}$

(c) We require $i = 0.500 I_i = \dfrac{0.500\mathcal{E}}{R}$, or

$$0.500\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}e^{-t/\tau} \rightarrow e^{-t/\tau} = 0.500$$

taking natural logarithm of both sides,

$$\ln(e^{-t/\tau}) = \ln(0.500) \rightarrow -\frac{t}{\tau} = -\ln(2.00)$$

where we have used the identity $\ln\left(\frac{1}{x}\right) = -\ln(x)$. Solving for the time $t$ gives

$$t = \tau(\ln(2.00)) = \frac{L}{R}(\ln(2.00)) = \frac{2.30\text{ H}}{16.0\ \Omega}(\ln(2.00)) = 0.0996\text{ s}$$

(d) In this case, $i = 0.100 I_i = \dfrac{0.100\mathcal{E}}{R}$ , or

$$0.100\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}e^{-t/\tau} \rightarrow e^{-t/\tau} = 0.100$$

taking natural logarithm of both sides,

$$\ln(e^{-t/\tau}) = \ln(0.100) \rightarrow -\frac{t}{\tau} = -\ln(10.0)$$

where we have used the identity $\ln\left(\frac{1}{x}\right) = -\ln(x)$. Solving for the time $t$ gives

$$t = \tau(\ln(10.0)) = \frac{L}{R}(\ln(10.0)) = \frac{2.30\text{ H}}{16.0\ \Omega}(\ln(10.0)) = 0.331\text{ s}$$

6 For a solenoid of length $\ell$, the inductance is $L = \frac{\mu_0 N^2 A}{\ell}$.

Thus, since $U_B = \frac{1}{2}Li^2 = \frac{\mu_0 N^2 A i^2}{2\ell}$, the stored energy is

$$U_B = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)(68)^2 \pi (6.00 \times 10^{-3}\ \text{m})^2 (0.770\ \text{A})^2}{2\ (0.080\ 0)\ \text{m}}$$

$$= \boxed{2.44 \times 10^{-6}\ \text{J}}$$

7 (a) The magnetic energy density is given by

$$u_B = \frac{B^2}{2\mu_0} = \frac{(4.50\ \text{T})^2}{2(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})} = 8.06 \times 10^6\ \text{J/m}^3 = \boxed{8.06\ \text{MJ}}$$

(b) The magnetic energy stored in the field equals $u$ times the volume of the solenoid (the volume in which $B$ is non-zero).

$$U_B = u_B V = (8.06 \times 10^6\ \text{J/m}^3)\left[(0.260\ \text{m})\pi(0.031\,0\ \text{m})^2\right]$$

$$= \boxed{6.32\ \text{kJ}}$$

8 This radio is a radiotelephone on a ship, according to frequency assignments made by international treaties, laws, and decisions of the National Telecommunications and Information Administration.

The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Thus, $C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{\left[2\pi(6.30 \times 10^6\ \text{Hz})\right]^2 (1.05 \times 10^{-6}\ \text{H})} = \boxed{608\ \text{pF}}$

9 (a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.100\ \text{H})(1.00 \times 10^{-6}\ \text{F})}} = \boxed{503\ \text{Hz}}$$

(b) The maximum charge on the capacitor is

$$Q = C\mathcal{E} = (1.00 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = \boxed{12.0\ \mu\text{C}}$$

(c) To find the maximum current $I_i$, we equate

$$\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_i^2$$

Then solve for $I_i$ to obtain

$$I_i = \mathcal{E}\sqrt{\frac{C}{L}} = 12.0\ \text{V}\sqrt{\frac{1.00 \times 10^{-6}\ \text{F}}{0.100\ \text{H}}} = \boxed{37.9\ \text{mA}}$$

(d) The total energy the circuit possesses at $t = 3.00$ s and at all times is

$$U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00 \times 10^{-6}\ \text{F})(12.0\ \text{V})^2 = \boxed{72.0\ \mu\text{J}}$$