Halliday ♦ Resnick ♦ Walker

# FUNDAMENTALS OF PHYSICS
# SIXTH EDITION

Selected Solutions

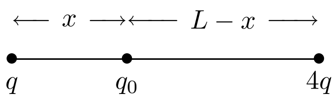
Chapter 22

22.9
22.27

9. (a) If the system of three charges is to be in equilibrium, the force on each charge must be zero. Let the third charge be $q_0$. It must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and $q_0$ could not be in equilibrium. Suppose $q_0$ is a distance $x$ from $q$, as shown on the diagram below. The force acting on $q_0$ is then given by

$$F_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_0}{x^2} - \frac{4qq_0}{(L-x)^2}\right)$$

where the positive direction is rightward. We require $F_0 = 0$ and solve for $x$. Canceling common factors yields $1/x^2 = 4/(L-x)^2$ and taking the square root yields $1/x = 2/(L-x)$. The solution is $x = L/3$.




The force on $q$ is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_0}{x^2} + \frac{4q^2}{L^2}\right) .$$

The signs are chosen so that a negative force value would cause $q$ to move leftward. We require $F_q = 0$ and solve for $q_0$:

$$q_0 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q$$

where $x = L/3$ is used. We now examine the force on $4q$:

$$\begin{aligned} F_{4q} &= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) \end{aligned}$$

which we see is zero. Thus, with $q_0 = -(4/9)q$ and $x = L/3$, all three charges are in equilibrium.

(b) If $q_0$ moves toward $q$ the force of attraction exerted by $q$ is greater in magnitude than the force of attraction exerted by $4q$. This causes $q_0$ to continue to move toward $q$ and away from its initial position. The equilibrium is unstable.

27. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where $a$ is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = (\sqrt{3}/2)a$. The force has magnitude

$$F = k\,\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})^2}{(3/4)(0.40\times10^{-9}\,\mathrm{m})^2} = 1.9\times10^{-9}\ \mathrm{N}\ .$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.