Halliday ♦ Resnick ♦ Walker

FUNDAMENTALS OF PHYSICS
SIXTH EDITION

Selected Solutions

Chapter 21

21.9
21.31
21.45

9. (a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi}) \implies T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w} .$$

The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$\begin{aligned} T_f &= \frac{(0.200\,\text{kg})(900\,\text{J/kg}\cdot\text{K})(100°\text{C}) + (0.0500\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(20°\text{C})}{(0.200\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) + (0.0500\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})} \\ &= 57.0°\text{C} \qquad \text{or} \quad 330\ \text{K} . \end{aligned}$$

(b) Now temperatures must be given in Kelvins: $T_{ai} = 393\,\text{K}$, $T_{wi} = 293\,\text{K}$, and $T_f = 330\,\text{K}$. For the aluminum, $dQ = m_a c_a\, dT$ and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} \\ &= (0.200\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) \ln\left(\frac{330\,\text{K}}{373\,\text{K}}\right) = -22.1\ \text{J/K} . \end{aligned}$$

(c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} \\ &= (0.0500\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) \ln\left(\frac{330\,\text{K}}{293\,\text{K}}\right) = +24.9\ \text{J/K} . \end{aligned}$$

(d) The change in the total entropy of the aluminum-water system is $\Delta S = \Delta S_a + \Delta S_w = -22.1\,\text{J/K} + 24.9\,\text{J/K} = +2.8\,\text{J/K}$.

31. (a) If T_H is the temperature of the high-temperature reservoir and T_L is the temperature of the low-temperature reservoir, then the maximum efficiency of the engine is

$$\varepsilon = \frac{T_\mathrm{H} - T_\mathrm{L}}{T_\mathrm{H}} = \frac{(800 + 40)\,\mathrm{K}}{(800 + 273)\ \mathrm{K}} = 0.78\ .$$

(b) The efficiency is defined by $\varepsilon = |W|/|Q_\mathrm{H}|$, where W is the work done by the engine and Q_H is the heat input. W is positive. Over a complete cycle, $Q_\mathrm{H} = W + |Q_\mathrm{L}|$, where Q_L is the heat output, so $\varepsilon = W/(W + |Q_\mathrm{L}|)$ and $|Q_\mathrm{L}| = W[(1/\varepsilon) - 1]$. Now $\varepsilon = (T_\mathrm{H} - T_\mathrm{L})/T_\mathrm{H}$, where T_H is the temperature of the high-temperature heat reservoir and T_L is the temperature of the low-temperature reservoir. Thus,

$$\frac{1}{\varepsilon} - 1 = \frac{T_\mathrm{L}}{T_\mathrm{H} - T_\mathrm{L}} \qquad \text{and} \qquad |Q_\mathrm{L}| = \frac{WT_\mathrm{L}}{T_\mathrm{H} - T_\mathrm{L}}\ .$$

The heat output is used to melt ice at temperature $T_i = -40°\mathrm{C}$. The ice must be brought to $0°\mathrm{C}$, then melted, so $|Q_\mathrm{L}| = mc(T_f - T_i) + mL_F$, where m is the mass of ice melted, T_f is the melting temperature ($0°$ C), c is the specific heat of ice, and L_F is the heat of fusion of ice. Thus, $WT_\mathrm{L}/(T_\mathrm{H} - T_\mathrm{L}) = mc(T_f - T_i) + mL_F$. We differentiate with respect to time and replace dW/dt with P, the power output of the engine, and obtain $PT_\mathrm{L}/(T_\mathrm{H} - T_\mathrm{L}) = (dm/dt)[c(T_f - T_i) + L_F]$. Thus,

$$\frac{dm}{dt} = \left(\frac{PT_\mathrm{L}}{T_\mathrm{H} - T_\mathrm{L}}\right)\left(\frac{1}{c(T_f - T_i) + L_F}\right)\ .$$

Now, $P = 100 \times 10^6\,\mathrm{W}$, $T_\mathrm{L} = 0 + 273 = 273\,\mathrm{K}$, $T_\mathrm{H} = 800 + 273 = 1073\,\mathrm{K}$, $T_i = -40 + 273 = 233\,\mathrm{K}$, $T_f = 0 + 273 = 273\,\mathrm{K}$, $c = 2220\,\mathrm{J/kg{\cdot}K}$, and $L_F = 333 \times 10^3\,\mathrm{J/kg}$, so

$$\begin{aligned}\frac{dm}{dt} &= \left[\frac{(100 \times 10^6\,\mathrm{J/s})(273\,\mathrm{K})}{1073\,\mathrm{K} - 273\,\mathrm{K}}\right]\left[\frac{1}{(2220\,\mathrm{J/kg \cdot K})(273\,\mathrm{K} - 233\,\mathrm{K}) + 333 \times 10^3\,\mathrm{J/kg}}\right] \\ &= 82\ \mathrm{kg/s}\ .\end{aligned}$$

We note that the engine is now operated between $0°\mathrm{C}$ and $800°\mathrm{C}$.

45. (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the right third, $n_C!$ are rearrangements of the n_C molecules in the center third, and $n_R!$ are rearrangements of the n_R molecules in the right third. These rearrangements do not produce a new configuration. Thus, the multiplicity is

$$W = \frac{N!}{n_L!\, n_C!\, n_R!} \ .$$

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!\,(N/2)!} \ .$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!\,(N/3)!\,(N/3)!} \ .$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!\,(N/2)!}{(N/3)!\,(N/3)!\,(N/3)!} \ .$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.16 \times 10^{16} \ .$$