Halliday ♦ Resnick ♦ Walker

FUNDAMENTALS OF PHYSICS
SIXTH EDITION

Selected Solutions

Chapter 17

17.25
17.31
17.39
17.49

25. (a) The displacement of the string is assumed to have the form $y(x,t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is $u(x,t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. For this wave the frequency is $f = 120\,\text{Hz}$ and the angular frequency is $\omega = 2\pi f = 2\pi(120\,\text{Hz}) = 754\,\text{rad/s}$. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}\,\text{m}$. The maximum speed is $u_m = (754\,\text{rad/s})(5.00 \times 10^{-3}\,\text{m}) = 3.77\,\text{m/s}$.

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin\theta$. Now θ is given by $\tan\theta = \partial y/\partial x = k y_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = k y_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1} \,.$$

Thus

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00 \times 10^{-3}\,\text{m}) = 0.138$$

and $\theta = 7.83°$. The maximum value of the transverse component of the tension in the string is $\tau_{\text{trans}} = (90.0\,\text{N}) \sin 7.83° = 12.3\,\text{N}$. We note that $\sin\theta$ is nearly the same as $\tan\theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau \partial y/\partial x = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is $u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}} \frac{\Delta y}{\Delta t} = \tau_{\text{trans}} u \,.$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is $(12.3\,\text{N})(3.77\,\text{m/s}) = 46.4\,\text{W}$.

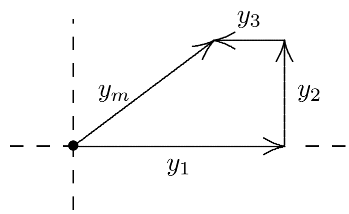
(e) As shown above $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50\,\text{cm}$.

31. (a) The phasor diagram is shown to the right: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave. The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1 \ .$$



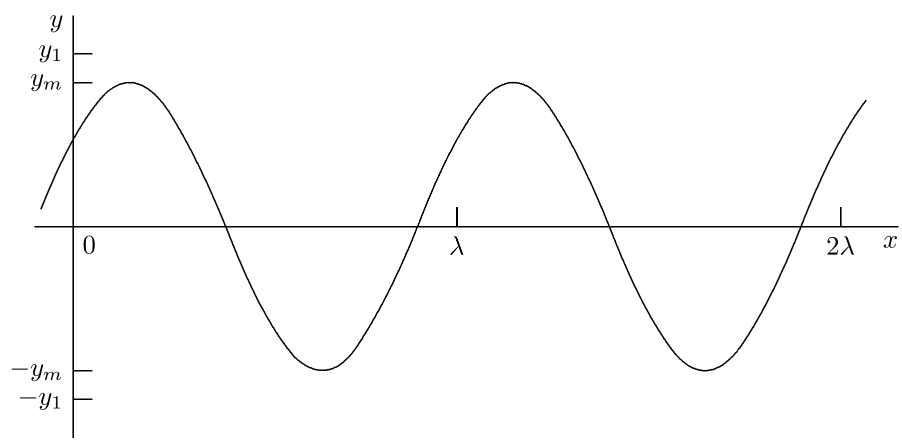
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\frac{y_{mv}}{y_{mh}} = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\frac{3}{4} = 0.644\,\text{rad} = 37^\circ \ .$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644\,\text{rad}) \ .$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



39. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n+1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n+1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n} .$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315\,\text{Hz}}{420\,\text{Hz} - 315\,\text{Hz}} = 3 .$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315\,\text{Hz})/3 = 105\,\text{Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then $v = \lambda f = 2Lf = 2(0.75\,\text{m})(105\,\text{Hz}) = 158\,\text{m/s}$.

49. We consider an infinitesimal segment of a string oscillating in a standing wave pattern. Its length is $\mathrm{d}x$ and its mass is $\mathrm{d}m = \mu\,\mathrm{d}x$, where μ is its linear mass density. If it is moving with speed u its kinetic energy is $\mathrm{d}K = \frac{1}{2}u^2\,\mathrm{d}m = \frac{1}{2}\mu u^2\,\mathrm{d}x$. If the segment is located at x its displacement at time t is $y = 2y_m \sin(kx)\cos(\omega t)$ and its velocity is $u = \partial y/\partial t = -2\omega y_m \sin(kx)\sin(\omega t)$, so its kinetic energy is

$$\mathrm{d}K = \left(\frac{1}{2}\right)\left(4\mu\omega^2 y_m^2\right)\sin^2(kx)\sin^2(\omega t) = 2\mu\omega^2 y_m^2 \sin^2(kx)\sin^2(\omega t)\ .$$

Here y_m is the amplitude of each of the traveling waves that combine to form the standing wave. The infinitesimal segment has maximum kinetic energy when $\sin^2(\omega t) = 1$ and the maximum kinetic energy is given by the differential amount

$$dK_m = 2\mu\omega^2 y_m^2 \sin^2(kx)\ .$$

Note that every portion of the string has its maximum kinetic energy at the same time although the values of these maxima are different for different parts of the string. If the string is oscillating with n loops, the length of string in any one loop is L/n and the kinetic energy of the loop is given by the integral

$$K_m = 2\mu\omega^2 y_m^2 \int_0^{L/n} \sin^2(kx)\,dx\ .$$

We use the trigonometric identity $\sin^2(kx) = \frac{1}{2}\left[1 + 2\cos(2kx)\right]$ to obtain

$$K_m = \mu\omega^2 y_m^2 \int_0^{L/n} \left[1 + 2\cos(2kx)\right]\mathrm{d}x = \mu\omega^2 y_m^2 \left[\frac{L}{n} + \frac{1}{k}\sin\frac{2kL}{n}\right]\ .$$

For a standing wave of n loops the wavelength is $\lambda = 2L/n$ and the angular wave number is $k = 2\pi/\lambda = n\pi/L$, so $2kL/n = 2\pi$ and $\sin(2kL/n) = 0$, no matter what the value of n. Thus,

$$K_m = \frac{\mu\omega^2 y_m^2 L}{n}\ .$$

To obtain the expression given in the problem statement, we first make the substitutions $\omega = 2\pi f$ and $L/n = \lambda/2$, where f is the frequency and λ is the wavelength. This produces $K_m = 2\pi^2\mu y_m^2 f^2\lambda$. We now substitute the wave speed v for $f\lambda$ and obtain $K_m = 2\pi^2\mu y_m^2 f v$.