Halliday ♦ Resnick ♦ Walker

FUNDAMENTALS OF PHYSICS
SIXTH EDITION

Selected Solutions

Chapter 16

16.29
16.39
16.53

29. (a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is $\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n+1)$, where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n+1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n+1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for k_1. The result is $k_1 = k(n+1)/n$.

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\,\Delta x = k_2\,\Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n+1)$, then solve for k_2. The result is $k_2 = k(n+1)$.

(c) To find the frequency when spring 1 is attached to mass m, we replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n+1)/n$ to obtain

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}\,f$$

where the substitution $f = (1/2\pi)\sqrt{k/m}$ was made.

(d) To find the frequency when spring 2 is attached to the mass, we replace k with $k(n+1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}\,f$$

where the same substitution was made.

39. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet-block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m+M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050\,\text{kg})(150\,\text{m/s})}{0.050\,\text{kg} + 4.0\,\text{kg}} = 1.85\,\text{m/s} .$$

When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance $\ell = (M+m)g/k$, somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M+m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m .$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$\begin{aligned} y_m &= \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)} \\ &= \sqrt{\frac{(0.050\,\text{kg} + 4.0\,\text{kg})(1.85\,\text{m/s})^2}{500\,\text{N/m}} - \frac{(0.050\,\text{kg})(9.8\,\text{m/s}^2)^2}{(500\,\text{N/m})^2}[2(4.0\,\text{kg}) + 0.050\,\text{kg}]} \\ &= 0.166\ \text{m} . \end{aligned}$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\,\text{kg})(150\,\text{m/s})^2 = 563\,\text{J}$. The kinetic energy of the bullet-block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050\,\text{kg} + 4.0\,\text{kg})(1.85\,\text{m/s})^2 = 6.94\ \text{J} .$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is $E/E_0 = (6.94\,\text{J})/(563\,\text{J}) = 0.0123$ or 1.23%.

53. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is $T = 2\pi/\omega = 2\pi\sqrt{I/C}$, where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved $(L/2)(1-\cos\theta)$ to the right. The length of the spring is now $\sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}$. If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\,\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 11–2. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}\ .$$