Halliday ♦ Resnick ♦ Walker

FUNDAMENTALS OF PHYSICS
SIXTH EDITION

Selected Solutions

Chapter 14

14.19
14.35
14.37
14.55

19. (a) The forces acting on an object being weighed are the downward force of gravity and the upward force of the spring balance. Let F_g be the magnitude of the force of Earth's gravity and let W be the magnitude of the force exerted by the spring balance. The reading on the balance gives the value of W. The object is traveling around a circle of radius R and so has a centripetal acceleration. Newton's second law becomes $F_g - W = mV^2/R$, where V is the speed of the object as measured in an inertial frame and m is the mass of the object. Now $V = R\omega \pm v$, where ω is the angular velocity of Earth as it rotates and v is the speed of the ship relative to Earth. We note that the first term gives the speed of a point fixed to the rotating Earth. The plus sign is used if the ship is traveling in the same direction as the portion of Earth under it (west to east) and the negative sign is used if the ship is traveling in the opposite direction (east to west).

Newton's second law is now $F_g - W = m(R\omega \pm v)^2/R$. When we expand the parentheses we may neglect the term v^2 since v is much smaller than $R\omega$. Thus, $F_g - W = m(R^2\omega^2 \pm 2R\omega v)/R$ and $W = F_g - mR\omega^2 \mp 2m\omega v$. When $v = 0$ the scale reading is $W_0 = F_g - mR\omega^2$, so $W = W_0 \mp 2m\omega v$. We replace m with W_0/g to obtain $W = W_0(1 \mp 2\omega v/g)$.

(b) The upper sign ($-$) is used if the ship is sailing eastward and the lower sign ($+$) is used if the ship is sailing westward.

35. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $-GMm/R + \frac{1}{2}mv^2 = 0$. We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m})} = 1.7 \times 10^3\ \text{m/s}\ .$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R+h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}\ .$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_gR + \frac{1}{2}v^2 = -\frac{a_gR^2}{(R+h)}\ .$$

The solution for h is

$$\begin{aligned} h &= \frac{2a_gR^2}{2a_gR - v^2} - R \\ &= \frac{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m})^2}{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m}) - (1000\,\text{m/s})^2} - (500 \times 10^3\,\text{m}) \\ &= 2.5 \times 10^5\ \text{m}\ . \end{aligned}$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R+h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2\ .$$

We substitute a_gR^2 for GM and cancel m, obtaining

$$-\frac{a_gR^2}{R+h} = -a_gR + \frac{1}{2}v^2\ .$$

The solution for v is

$$\begin{aligned} v &= \sqrt{2a_gR - \frac{2a_gR^2}{R+h}} \\ &= \sqrt{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m}) - \frac{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m})^2}{500 \times 10^3\,\text{m} + 1000 \times 10^3\,\text{m}}} \\ &= 1.4 \times 10^3\ \text{m/s}\ . \end{aligned}$$

37. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2 \ .$$

The solution for v is

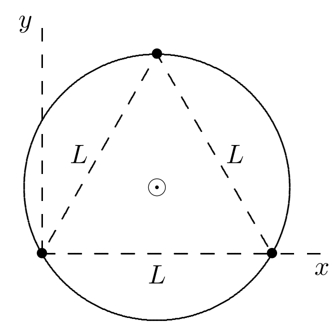
$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\,\text{kg})}{10^{10}\,\text{m}}} = 8.2 \times 10^4\,\text{m/s} \ .$$

(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes $-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$\begin{aligned} v &= \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} \\ &= \sqrt{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\,\text{kg})\left(\frac{1}{2 \times 10^5\,\text{m}} - \frac{1}{10^{10}\,\text{m}}\right)} \\ &= 1.8 \times 10^7\ \text{m/s} \ . \end{aligned}$$

55. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.

The stars rotate about their center of mass (marked by $\odot$ on the diagram to the right) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $(\sqrt{3}/2)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The x coordinate of the center of mass is $x_c = (L + L/2)/3 = L/2$ and the y coordinate is $y_c = (\sqrt{3}L/2)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is $R = \sqrt{x_c^2 + y_c^2} = \sqrt{(L^2/4) + (L^2/12)} = L/\sqrt{3}$.



Once the substitution for R is made Newton's second law becomes $(2GM^2/L^2)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified somewhat by recognizing that $\cos 30° = \sqrt{3}/2$, and we divide the equation by M. Then, $GM/L^2 = v^2/L$ and $v = \sqrt{GM/L}$.