Halliday ◆ Resnick ◆ Walker

FUNDAMENTALS OF PHYSICS
SIXTH EDITION

Selected Solutions

<u>Chapter 12</u>

12.37
12.45

37. Suppose cylinder 1 exerts a uniform force of magnitude F on cylinder 2, tangent to the cylinder's surface at the point of contact. The torque applied to cylinder 2 is $\tau_2 = R_2 F$ and the angular acceleration of that cylinder is $\alpha_2 = \tau_2/I_2 = R_2 F/I_2$. As a function of time its angular velocity is

$$\omega_2 = \alpha_2 t = \frac{R_2 F t}{I_2} \ .$$

The forces of the cylinders on each other obey Newton's third law, so the magnitude of the force of cylinder 2 on cylinder 1 is also F. The torque exerted by cylinder 2 on cylinder 1 is $\tau_1 = R_1 F$ and the angular acceleration of cylinder 1 is $\alpha_1 = \tau_1/I_1 = R_1 F/I_1$. This torque slows the cylinder. As a function of time, its angular velocity is $\omega_1 = \omega_0 - R_1 F t/I_1$. The force ceases and the cylinders continue rotating with constant angular speeds when the speeds of points on their rims are the same ($R_1 \omega_1 = R_2 \omega_2$). Thus,

$$R_1 \omega_0 - \frac{R_1^2 F t}{I_1} = \frac{R_2^2 F t}{I_2} \ .$$

When this equation is solved for the product of force and time, the result is

$$F t = \frac{R_1 I_1 I_2}{I_1 R_2^2 + I_2 R_1^2} \omega_0 \ .$$

Substituting this expression for Ft in the ω_2 equation above, we obtain

$$\omega_2 = \frac{R_1 R_2 I_1}{I_1 R_2^2 + I_2 R_1^2} \omega_0 \ .$$

45. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is $= I\omega\hat{k} = -MR^2|\omega|\hat{k}$, where $\hat{k}$ is *up* in Fig. 12-40 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 12-40 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m(v-|\omega|R)R\hat{k}$. Conservation of angular momentum yields

$$0 = -MR^2|\omega|\,\hat{k} + m\,(v - |\omega|R)\,R\,\hat{k}\;.$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{mv}{(M+m)R}\;.$$