CHAPTER 8 HINT FOR EXERCISE 4

(a), (b), and (c) Take the y axis to be positive in the upward direction. Then the work done by the gravitational force as the roller coaster goes from a place with y coordinate y_i to a place with y coordinate y_f is given by

$$W = -mg(y_f - y_i)\,,$$

where m is the mass of the roller coaster. The work is negative if the final position is above the initial position ($y_f > y_i$) and is positive if the final position is below the initial position ($y_f < y_i$). For point A, $y_f - y_i = 0$; for point B, $y_f - y_i = -h/2$; for point C, $y_f - y_i = -h$.

(d) and (e) If the gravitational potential energy is taken to be zero at a place with the y coordinate of the roller coaster is y_i, then the potential energy when it is at a place where the y coordinate is y_f is given by

$$U = -W = mg(y_f - y_i)\,.$$

This is positive is $y_f > y_i$ and negative if $y_f < y_i$. Take $y = 0$ at point C. Then for point B, $y_f - y_i = h/2$ and for point A, $y_f - y_i = h$.

(f) The gravitational potential energy is directly proportional to the mass of the roller coaster.

Chapter 8 Hint for Problem 6

(a) and (b) Multiply the gravitational force on the block by the vertical distance the block travels to each of the points. In (a) it starts a distance $5R$ above the bottom of the loop and ends a distance R above the bottom. In (b) it starts at the same place but ends a distance $2R$ above the bottom of the loop.

(c), (d), and (e) The potential energy when the block is a vertical distance h above the point where the potential energy is zero is given by $U = mgh$. Calculate h for each of the given points.

(f) Recall that the gravitational potential energy depends only on the initial and final positions of objects in the system.

(a) Take the y axis to be positive in the upward direction. Then the work done by the force of gravity as the ball goes from a place with y coordinate y_i to a place with y coordinate y_f is given by

$$W = -mg(y_f - y_i)\,,$$

where m is the mass of the ball. When the ball is at its lowest point it is a distance L below the pivot point. When it is at the position shown on Fig. 8–29 it is a vertical distance $L\cos\theta$ below the pivot point. Thus

$$y_f - y_i = mgL - mgL\cos\theta = mgL(1 - \cos\theta)\,.$$

(b) The change in the gravitational potential energy of the Earth-ball system is the negative of the work done by the force of gravity:

$$\Delta U = -W = -mgL(1 - \cos\theta)\,.$$

(c) The gravitational potential energy decreases as the ball falls. If it is zero at the lowest point then it must be greater at the release point and it is greater by $mgL(1 - \cos\theta)$.

(d) As θ increases, $\cos\theta$ decreases and $1 - \cos\theta$ increases.

Chapter 8 Hint for Problem 14

(a) Two forces act on the ball, the force of gravity and the force of the rod. The force of the rod does no work since it is perpendicular to the path of the ball. The gravitational force is conservative. The mechanical energy of the ball-Earth system is conserved.

Take the gravitational potential energy to be zero when the ball is at the bottom of its swing. The initial potential energy is then $mgL(1 - \cos\theta)$, the initial kinetic energy is zero, the final potential energy is zero, and the final kinetic energy is $\frac{1}{2}mv^2$, where v is the speed of the ball at the bottom of its swing. Conservation of mechanical energy yields

$$mgL(1 - \cos\theta) = \frac{1}{2}mv^2\,.$$

Solve for v.

(b) Notice that the mass cancels from the conservation of mechanical energy equation.

(a) Take the elastic potential energy to be zero when the spring is relaxed. The potential energy when it is compressed is then given by

$$U = \frac{1}{2}kx^2\,,$$

where k is the spring constant and x is the extent of the compression.

(b) The incline is frictionless, the normal force does zero work, and gravity is a conservative force. The mechanical energy of the block-spring-Earth system is conserved. This energy is the sum of the kinetic energy of the block, the gravitational potential energy U_g of the block and Earth, and the elastic potential energy U_e of the block and spring.

The kinetic energy of the block is zero at the beginning and end of the interval so conservation of mechanical energy yields

$$\Delta U_g + \Delta U_e = 0\,.$$

Use $\Delta U_e = -\frac{1}{2}kx^2$ to shown that $\Delta U_g = +\frac{1}{2}kx^2$.

(c) The change in the gravitational potential energy is given by $\Delta U_g = mgh$, where m is the mass of the block and h is the vertical height of the highest point. If d is the distance along the incline, then $d \sin\theta = h$, where θ is the angle of the incline.

Solve $mgd \sin\theta = \frac{1}{2}kx^2$ for d.

[ans: (a) 39.2 J; (b) 39.2 J; (c) 4.00 m]

Two forces act on Tarzan: the force of gravity mg, down, and the force of the vine T, inward along the line from Tarzan to the center of his circular path. Here m is Tarzan's mass. The combination provides the centripetal force that keeps him on his circular path. The force of the vine has its greatest magnitude when Tarzan is at the bottom of his swing. Then the force of the vine is upward. Calculate the force of vine when Tarzan is at the low point and see if it exceeds 950 N.

Take the upward direction to be positive. At the bottom of the swing the net force on Tarzan is $T - mg$. At this point Tarzan's acceleration is upward. He is not speeding up or slowing down. Thus the magnitude of Tarzan's acceleration is $a = v^2/L$, where v is his speed at the bottom of the swing and L is the length of the vine (and the radius of Tarzan's circular path). Newton's second law yields

$$T - mg = \frac{mv^2}{L}\,.$$

To calculate the force of the vine you need to know Tarzan's speed at the bottom of the swing. Use conservation of mechanical energy. The only force that does work on Tarzan is the gravitational force and this force is conservative. Note that the vine is always perpendicular to Tarzan's velocity and therefore does zero work. Take the gravitational potential energy to be zero when Tarzan is on the cliff. Then the gravitational potential energy when he is at the bottom of the swing is $U_f = -mgd$, where d is vertical distance of his descent (3.2 m). His initial kinetic energy is zero and his final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is his speed at the bottom of the swing. Solve

$$0 = \frac{1}{2}mv^2 - mgd$$

for v^2.

Now go back to $T - mg = mv^2/L$ and solve for T.

The only force that does work on the ball is the force of gravity and this force is conservative. The string is always perpendicular to the velocity of the ball and so does zero work. Thus mechanical energy is conserved and you may use it to compute the speed of the ball when it is at various positions.

Take the y axis to be vertical with up being the positive direction and place the origin at the pivot point of the pendulum. Take the gravitational potential energy of the ball-Earth system to zero when the y coordinate of the ball is zero. Then the gravitational potential energy when the ball has coordinate y is $U = mgy$, where m is the mass of the ball.

When the string makes the angle θ with the vertical the ball is a vertical distance $L\cos\theta$ below the pivot point and the gravitational potential energy is $U = -mgL\cos\theta$. Let θ_i $(= 30.0^\circ)$ be the initial angle of the pendulum and θ_f be its final angle (at the end of some interval). When $\theta = \theta_i$ the kinetic energy is zero. Write the kinetic energy for some other angle as $\frac{1}{2}mv^2$, where v is the speed of the ball (an unknown quantity). Conservation of mechanical energy yields

$$-mgL\cos\theta_i = \frac{1}{2}mv^2 - mgL\cos\theta\,.$$

(a) Put $\theta = 20.0^\circ$ and solve for v.

(b) The maximum speed occurs when $\cos\theta$ has its greatest value. This is when θ has its least value and the least value is zero. Put θ equal to zero and solve for v.

(c) Put v equal to one-third of the value you found in part (b) and solve for $\cos\theta$, then for θ.

The only forces that do work on the canister are the force of gravity and the spring force. Both of these are conservative, so the mechanical energy of the canister-Earth-spring system is conserved. Note that the normal force of the incline on the canister is perpendicular to the displacement of the canister and so does zero work.

Take the gravitational potential energy to zero when the canister is at the bottom of the incline. When the canister is a distance d up the incline (as measured along the incline) then it is a vertical distance $d \sin 37.0^\circ$ above the bottom and the gravitational potential energy is $U_g = mgd \sin 37.0^\circ$, where m is the mass of the canister.

Take the elastic potential energy of the spring to be zero when the spring is in its relaxed state. When the spring is compressed (or expanded) by x the elastic potential energy is $U_e = \frac{1}{2}kx^2$, where k is the spring constant.

(a) Initially the canister is $d_i = 1.20\,\mathrm{m}$ from the bottom of the incline and has zero kinetic energy. When it leaves the spring it is $d_f = 1.00\,\mathrm{m}$ from the bottom of the incline and its kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is its speed then. The initial compression of the spring is $x_i = 0.200\,\mathrm{m}$ and the final compression is $x_f = 0$. Conservation of mechanical energy yields

$$mgd_i \sin 37.0^\circ + \frac{1}{2}kx_i^2 = \frac{1}{2}mv^2 + mgd_f \sin 37.0^\circ\,.$$

Solve for v.

(b) Now set d_f equal to zero and solve for v.

Chapter 8 Hint for Problem 36

(a) Since the force and potential energy are related by

$$F = -\frac{\mathrm{d}U}{\mathrm{d}x},$$

you want to calculate the slope of the curve at $x = 2.0\,\mathrm{m}$. The force is the negative of the value you find.

(b) First calculate the mechanical energy E_{mec}. Add the potential and kinetic energies when the particle is at $x = 2.0\,\mathrm{m}$. Read the potential energy from the graph and calculate the kinetic energy using $K = \frac{1}{2}mv^2$. Now draw a horizontal line across the graph at $U = E_{\mathrm{mec}}$ and note the values of x where it crosses the curve. At these points the kinetic energy is 0 and the particle reverses its direction of motion.

(c) The mechanical energy is

$$E_{\mathrm{mec}} = \frac{1}{2}mv^2 + U.$$

Read the value of U at $x = 7.0\,\mathrm{m}$ from the graph and us the conservation law calculate v.

The thermal energy of the floor and cube together increases by fd, where f is the frictional force of the floor on the cube and d is the distance the cube is pushed. Since the cube moves with constant speed, Newton's second law tells us that the net force on the cube is zero. Since the only two horizontal forces that push on the cube are the applied force and the frictional force, these two forces have the same magnitude (and are in opposite directions). Substitute $f = 15\,\mathrm{N}$ and $d = 3.0\,\mathrm{m}$ into $\Delta E_{\mathrm{th}} = fd$ to find the total increase in thermal energy. The increase in the thermal energy of the floor is this result minus $20\,\mathrm{J}$.

The worker's force is constant so the work is does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement of the block. The force makes an angle of 32° with the horizontal, so $W = Fd\cos 32°$. You need to find the magnitude F of the worker's force.

Use Newton's second law. Draw a free body diagram for the block. Four forces act on it: the worker's force F, at an angle of 32° below the horizontal, the force of gravity mg, down, the normal force N of the floor, up, and the frictional force f, opposite the direction of motion. Take the x axis to be horizontal and positive in the direction of motion. Take the y axis to be vertical and in the direction of the normal force. Then, since the acceleration of the block is zero, the x component of Newton's second law is

$$F\cos 32° - \mu_k N = 0$$

and the y component is

$$N - F\sin 32° - mg = 0\,.$$

Here $\mu_k N$, where μ_k is the coefficient of kinetic friction, was used for the frictional force. Solve the y component equation for N and use the result to substitute for N in the x component equation, then solve for F.

(b) The worker's force does work on the block but the kinetic energy of the block does not increase. All the work done by the worker's force must end up as thermal energy in the block and floor.

Chapter 8 Hint for Exercise 46

Calculate the initial and final mechanical energies and find their difference. Take the system to consist of the Frisbee and Earth and include both kinetic and potential energies. Use $U = mgh$, where h is the height of the Frisbee above the ground and m is the mass of the Frisbee, to compute the potential energy. Use $K = \frac{1}{2}mv^2$, where v is the speed of the Frisbee, to compute the kinetic energy.

Chapter 8 Hint for Problem 50

Initially the mechanical energy of the block-spring system is entirely elastic potential energy. The block has zero kinetic energy. Thus the mechanical energy is initially $E_{\text{mec }i} = \frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression of the spring. After the block has come to rest on the table top all this energy has been converted to thermal energy in the block and table top. In terms of the frictional force f and the length d of the slide, the increase in total thermal energy (and the decrease in mechanical energy) is given by $\Delta E_{\text{th}} = fd$. Now $f = \mu_k N$, where μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force of the table top on the block. Since the table top is horizontal $N = mg$, where m is the mass of the block. Thus $\Delta E_{\text{th}} = \mu_k mgd$. Equate this to $\frac{1}{2}kx^2$ and solve for μ_k.

(a) Take the system to consist of the crate, the ramp, and Earth and let d be the distance the crate slides down the ramp. The gravitational potential energy changes by $\Delta U = -mgh$ and the kinetic energy changes by $\Delta K = \frac{1}{2}mv^2$, where h is the original height of the crate above the factory floor and v is its speed at the bottom of the ramp. The force of friction changes the mechanical energy by $-fd$ so

$$-mgh + \frac{1}{2}mv^2 = -fd\,.$$

A little geometry shows that $h = d\sin\theta$, where θ is the angle between the ramp and the horizontal. Furthermore,

$$f = \mu_k N = \mu_k mg\cos\theta\,,$$

where N $(= mg\cos\theta)$ is the normal force of the ramp on the crate. Solve

$$-mgd\sin\theta + \frac{1}{2}mv^2 = -\mu_k mgd\cos\theta$$

for v.

(b) As it starts across the floor its kinetic energy is $\frac{1}{2}mv^2$, where v is its speed at the bottom of the ramp (the answer to a). When it stops sliding its kinetic energy is zero. Because the floor is horizontal the potential energy does not change. If the crate slides a distance d across the floor, the mechanical energy changes by $-fd = -\mu_k Nd = -\mu_k mgd$. Thus

$$-\frac{1}{2}mv^2 = -\mu_k mgd\,.$$

Solve for d.

(c) Notice that the mass cancels from the equations you used.