Chapter 4 Hint for Problem 4

Place the origin of a coordinate system at the radar, with the x axis running from west to east and the y axis running upward. Let $\vec{r}_1$ be the position vector from the radar to the plane at the first observation and let $\vec{r}_2$ be the position vector from the radar to the plane at the last observation. The displacement vector is given by $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. Find the components of $\vec{r}_1$ and $\vec{r}_2$ and use them to calculate the components of $\Delta\vec{r}$.

The x component of $\vec{r}_1$ is $x_1 = (360\,\mathrm{m})\cos 40^\circ$ and the y component is $y_1 = (360\,\mathrm{m})\sin 40^\circ$. The x component of $\vec{r}_2$ is $x_2 = (790\,\mathrm{m})\cos 163^\circ$ and the y component is $y_2 = (790\,\mathrm{m})\sin 163^\circ$. The x component of the displacement is given by $\Delta x = x_2 - x_1$ and the y component is given by $\Delta y = y_2 - y_1$.

(a) Take the x axis to run from west to east and the y axis to run from south to north. Put the origin at oasis A. Then the first displacement is

$$\vec{r}_1 = (r_1 \cos 37°)\,\hat{\imath} + (r_1 \sin 37°)\,\hat{\jmath}$$

and the second is

$$\vec{r}_2 = -r_2\,\hat{\jmath}\,,$$

where r_1 and r_2 are the magnitudes. The total displacement from oasis A to the resting place is $\vec{r}_{\text{rest}} = \vec{r}_1 + \vec{r}_2$. Calculate the components of $\vec{r}_{\text{rest}}$, then use $r_{\text{rest}} = \sqrt{r^2_{\text{rest}\ x} + r^2_{\text{rest}\ y}}$ and $\tan\theta_{\text{rest}} = r_{\text{rest}\ y}/r_{\text{rest}\ x}$ to find the magnitude r_{rest} and the angle θ_{rest} with the positive x axis.

(b) The average velocity is

$$\vec{v}_{\text{avg}} = \vec{r}_{\text{rest}}/\Delta t\,,$$

where Δt is the total time, including the resting time.

(c) The average speed is the distance traveled divided by the total time. The distance traveled is the sum of the magnitudes of the two displacements: $d = r_1 + r_2$. Note that this is not the same as the magnitude of the total displacement.

(d) The average velocity required for the remainder of the trip to oasis B is the displacement from the resting place to oasis B divided by the time remaining. The displacement of oasis B relative to oasis A is $\vec{r}_B = r_B\,\hat{\imath}$ and its displacement from the resting place is $\vec{r}_B - \vec{r}_{\text{rest}}$. The time remaining is $120\,\text{h} - 50\,\text{h} - 35\,\text{h} - 5\,\text{h} = 30\,\text{h}$.

(a), (b), and (c) To find the position vector, substitute $t = 2.00\,\mathrm{s}$ into the expression for $\vec{r}$. To find the velocity, differentiate the expression for $\vec{r}$ with respect to time, then substitute $t = 2.00\,\mathrm{s}$. To find the acceleration, differentiate the expression for the velocity with respect to time, then substitute $t = 2.00\,\mathrm{s}$.

(d) The velocity vector is tangent to the path so the slope of the line is given by v_y/v_x. This is the tangent of the angle between the line and the positive x axis.

$\big[$ ans: (a) $(6.00\,\mathrm{m})\,\hat{\imath} - (106\,\mathrm{m})\,\hat{\jmath}$; (b) $(19.0\,\mathrm{m/s})\,\hat{\imath} - (224\,\mathrm{m/s})\,\hat{\jmath}$; (c) $(24.0\,\mathrm{m/s^2})\,\hat{\imath} - (336\,\mathrm{m/s^2})\,\hat{\jmath}$; (d) 85.2° below the x axis $\big]$

(a) Differentiate the expression for $\vec{v}$ to find an expression for the acceleration $\vec{a}$. Evaluate it for $t = 3.0\,\mathrm{s}$.

(b) Set the expression for $\vec{a}$ equal to 0 and solve for t.

(c) Set the expression for $\vec{v}$ equal to 0 and solve for t. For the velocity to be zero both components must vanish for the same value of t.

(d) Develop an expression for the speed as a function of time. It is the square root of the sum of the squares of the velocity components. Set the expression equal to $10\,\mathrm{m/s}$ and solve for t.

You should obtain $\sqrt{(6.0t - 4.0t^2)^2 + 8.0^2} = 10$, where units have been omitted. Square both sides and obtain $(6.0t - 4.0t^2)^2 = 36$. Take the square root of both sides to obtain $6.0t - 4.0t^2 = \pm 6$. The choice of one sign will lead to an imaginary solution. Use the other and solve the quadratic equation for t. One solution is negative and is not allowed by the conditions of the problem. The other solution is the correct solution.

You want to solve for the value of θ such that the position vectors of the two particles are the same at some instant of time.

The position vector of particle A is given by

$$\vec{r}_A = vt\,\hat{\imath} + d\,\hat{\jmath}\,,$$

where $d = 30\,\mathrm{m}$. The position vector of particle B is given by

$$\vec{r}_B = \frac{1}{2}a_x t^2\,\hat{\imath} + \frac{1}{2}a_y t^2\,\hat{\jmath} = \frac{1}{2}at^2\sin\theta\,\hat{\imath} + \frac{1}{2}at^2\cos\theta\,\hat{\jmath}\,,$$

where $a\sin\theta$ was substituted for a_x and $a\cos\theta$ was substituted for a_y. A collision occurs if $\frac{1}{2}at^2\sin\theta = vt$ and $\frac{1}{2}at^2\cos\theta = d$. Solve the first equation for t and substitute the resulting expression into the second. You should obtain $2v^2\cos\theta = ad\sin^2\theta$, an equation to be solved for θ. Use the trigonometric identity $\sin^2\theta = 1 - \cos^2\theta$ to obtain an equation that contains only $\cos\theta$. It is $\cos^2\theta + (2v^2/ad)\cos\theta - 1 = 0$. Solve this quadratic equation for $\cos\theta$ and the result for θ.

CHAPTER 4 HINT FOR EXERCISE 20

(a) Place the origin of the coordinate system at the throwing point and take the time to be 0 when the dart is thrown. The y axis is vertically upward and the x axis is horizontal. Let x and y be the coordinates of point Q and let t be the time the dart hits there. The vertical component of the initial velocity is 0 so $y = -\frac{1}{2}gt^2$. Evaluate this expression.

(b) Evaluate $x = v_{0x}t$.

(a) Place the coordinate system with its origin at the release point of the ball. Take the y axis to be positive in the upward direction and the x axis to be horizontal in the plane of the ball's motion. Take the time to be 0 at the instant the ball is released. Assume the ball hits the wall before it hits the ground and take x and y to be the coordinates of the point where it hits and t to be the time when it hits. You already know $x = 22.0\,\mathrm{m}$, $v_{0x} = 25.0\cos 40^\circ$, and $v_{0y} = 25.0\sin 40^\circ$. Use $x = v_{0x}t$ to find t, then evaluate $y = v_{0y}t - \frac{1}{2}gt^2$.

(b) Evaluate $v_x = v_{0x}$ and $v_y = v_{0y} - gt$.

(c) Note the sign of v_y, the vertical component of the velocity when the ball hits the wall. If the result is positive, the ball has not yet reached its highest point. If the result is negative, it has.

Chapter 4 Hint for Problem 30

(a) and (b) Take the x axis to be horizontal and the y axis to be vertical with the positive direction upward. Put the origin at the launch point. Then the x coordinate of the projectile is given by $x = v_{0x}t$ and the y coordinate is given by $y = v_{0y}t - \frac{1}{2}gt^2$. Solve for v_{0x} and v_{0y}.

(c) Solve $v_y = v_{0y} - gt$ for the time t when $v_y = 0$. This is the time the projectile reaches the highest point on its trajectory. The horizontal component of its displacement then is $x = v_{0x}t$.

Chapter 4 Hint for Problem 36

Take the x axis to be horizontal and in the direction of the kick; take the y axis to be vertically upward; place the origin at the place where the ball is kicked and set the time t equal to zero at the time of the kick. Then the y coordinate of the ball is given by $y = v_{0y}t - \frac{1}{2}gt^2$, where $v_{0y} = v_0 \sin 45°$ and $v_0 = 19.5\,\mathrm{m/s}$. The ball lands when $y = 0$ for the second time. (The first time $y = 0$ is $t = 0$, the time of the kick.) That is, it lands when $t = 2v_{0y}/g$. Its x coordinate then is $x = v_{0x}t$, where $v_{0x} = v_0 \cos 45°$. The distance the player must run is $d = 55\,\mathrm{m} - x$ and he must do it in the time t. His average speed must be d/t.

Chapter 4 Hint for Exercise 42

The sprinter has an acceleration because the direction of his velocity is changing with time. The magnitude is given by $a = v^2/r$. Evaluate this expression.

The distance is equal to the circumference of the circular path of the fan tip: $d = 2\pi r$, where r is the radius of the path. The time for one revolution is $T = (60\,\mathrm{s})/(1200\,\mathrm{rev/s})$. This is the period of the motion. The speed of the tip is $v = d/T$. The tip is in uniform circular motion. Its acceleration is a centripetal acceleration and has a magnitude that is given by $a = v^2/r$.

(a) Solve $a = v^2/r$ for r. The acceleration is $a = 0.050(9.8\,\mathrm{m/s^2}) = 0.49\,\mathrm{m/s^2}$. Convert the speed to meters per second.

(b) Solve $a = v^2/r$ for v.

(a), (b), and (c) Let θ be the angle between the radial line from the center of the circle to particle and the negative y direction and take it to be positive in the counterclockwise direction. Then the position vector of the particle is

$$\vec{r} = -r\,\hat{\jmath} + (r\cos\theta)\,\hat{\imath} - (r\cos\theta)\,\hat{\jmath} = (r\sin\theta)\,\hat{\imath} - r(1+\cos\theta)\,\hat{\jmath}\,.$$

The angle ϕ that the position vector from O to the particle makes with the positive x axis is given by $\tan\phi = r_y/r_x$.

At $t = 5.00\,\mathrm{s}$ the particle has gone one-quarter of the way around, so $\theta = 90^\circ$. At $t = 7.50\,\mathrm{s}$ the particle has gone 45° further, so $\theta = 135^\circ$. At $t = 10.0\,\mathrm{s}$ the particle has gone half of the way around, so $\theta = 180^\circ$.

(d) The particle's displacement is $\Delta\vec{r} = \vec{r}_{10} - \vec{r}_5$, where $\vec{r}_5$ is the position vector at $t = 5.00\,\mathrm{s}$ and $\vec{r}_{10}$ is the position vector at $t = 10.0\,\mathrm{s}$.

(e) The average velocity is $\vec{v}_{\mathrm{avg}} = \Delta\vec{r}/\Delta t$, where $\Delta t = 5.00\,\mathrm{s}$.

(f) and (g) At the beginning of the interval the particle is going in the positive y direction and at the end of the interval it is going in the negative x direction. It speed is $v = 2\pi r/T$, where r is the radius of the circular path and T is the time for one revolution.

(h) and (i) The magnitude of the acceleration is $a = v^2/r$ at all times. The acceleration vector points from the particle toward the center of the circle. At $t = 5.00\,\mathrm{s}$ it is in the negative x direction and at $t = 10.0\,\mathrm{s}$ it is in the negative y direction.

Chapter 4 Hint for Exercise 56

Take the x axis to be along the path of the player, with the forward direction positive and let the y axis be horizontal and perpendicular to the x axis. The velocity $\vec{v}_{BG}$ of the ball relative to the ground is given by

$$\vec{v}_{BG} = \vec{v}_{BP} + \vec{v}_{PG}\,,$$

where $\vec{v}_{BP}$ is the velocity of the ball relative to the player and $\vec{v}_{PG}$ is the velocity of the player relative to the ground. You want to find $v_{BP\,x}$ and $v_{BP\,y}$ so that $v_{BG\,x} = 0$. Thus

$$0 = v_{BP\,x} + v_{PG\,x}$$

and

$$v_{BP\,x} = -v_{PB\,x}\,.$$

Use $v_{BP}^2 = v_{BP\,x}^2 + v_{BP\,y}^2$ to find $v_{BP\,y}$. The angle θ you want is given by $\tan\theta = v_{BP\,y}/v_{BP\,x}$. Don't forget that $v_{BP\,x}$ is negative and that you may need to add 180° to the angle given by your calculator.