(a) Evaluate

$$\Phi = \int \vec{E} \cdot \mathrm{d}\vec{A}\,.$$

For the right face, $\mathrm{d}\vec{A} = \mathrm{d}A\,\hat{\jmath}$. In (a) $\vec{E} \cdot \mathrm{d}\vec{A} = 0$, in (b) $\vec{E} \cdot \mathrm{d}\vec{A} = -2.00\,\mathrm{d}A$, and in (c) $\vec{E} \cdot \mathrm{d}\vec{A} = 0$.

(d) All the given fields are uniform. Whatever flux is through one face, flux of exactly the same magnitude but opposite sign is through the opposite face.

[ans: (a) 0; (b) $-3.92\,\mathrm{N \cdot m^2/C}$; (c) 0; (d) 0 for each field]

Use $\Phi = \oint \vec{E} \cdot \mathrm{d}\vec{A}$.

(a) On the right face

$$\vec{E} \cdot \mathrm{d}\vec{A} = (3.00y\,\hat{\jmath}) \cdot (\mathrm{d}x\,\mathrm{d}z\,\hat{\jmath})\,,$$

with $y = a$, the length of a cube edge. The contribution of this face to the electric flux is $(3.00a)(a^2) = 3.00a^3$. On the left face

$$\vec{E} \cdot \mathrm{d}\vec{A} = (3.00y\,\hat{\jmath}) \cdot (-\mathrm{d}x\,\mathrm{d}z\,\hat{\jmath})\,,$$

with $y = 0$, so the contribution of this face is 0. On the other faces, the field is parallel to the area (perpendicular to the area vector) so their contributions are 0.

(b) On the right face

$$\vec{E} \cdot \mathrm{d}\vec{A} = [-4.00\,\hat{\imath} + (6.00 + 3.00y)\,\hat{\jmath}] \cdot (\mathrm{d}x\,\mathrm{d}z\,\hat{\jmath})\,,$$

with $y = a$. The contribution of this face to the electric flux is $(6.00 + 3.00a)(a^2)$. On the left face

$$\vec{E} \cdot \mathrm{d}\vec{A} = [-4.00\,\hat{\imath} + (6.00 + 3.00y)\,\hat{\jmath}] \cdot (-\mathrm{d}x\,\mathrm{d}z\,\hat{\jmath})\,,$$

with $y = 0$, so the contribution of this face is $-6.00a^2$. On the front face

$$\vec{E} \cdot \mathrm{d}\vec{A} = [-4.00\,\hat{\imath} + (6.00 + 3.00y)\,\hat{\jmath}] \cdot (\mathrm{d}y\,\mathrm{d}z\,\hat{\imath}) = -4.00\,\mathrm{d}y\,\mathrm{d}z\,,$$

so the contribution of this face is $-4.00a^2$. On the back face

$$\vec{E} \cdot \mathrm{d}\vec{A} = [-4.00\,\hat{\imath} + (6.00 + 3.00y)\,\hat{\jmath}] \cdot (-\mathrm{d}y\,\mathrm{d}z\,\hat{\imath}) = 4.00(\mathrm{d}y\,\mathrm{d}z)\,,$$

so the contribution of this face is $4.00a^2$. The top and bottom faces make no contributions.

(c) Use $q = \epsilon_0\Phi$.

[ans: (a) $8.23\,\mathrm{N \cdot m^2/C}$; (b) $8.23\,\mathrm{N \cdot m^2/C}$; (c) $72.8\,\mathrm{pC}$ in each case]

CHAPTER 24 HINT FOR EXERCISE 14

(a) The surface charge density is the charge divided by the surface area:

$$\sigma = \frac{q}{4\pi R^2} ,$$

where R is the radius of the satellite.

(b) Use $E = \sigma/\epsilon_0$.

Use Gauss' law:

$$\epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{A} = q_{\mathrm{enc}} \,,$$

where the integral is over a Gaussian surface and q_{enc} is the net charge enclosed by the Gaussian surface. For each part take the Gaussian surface to be a cylinder of radius r and length L that is concentric with the metal tube. The electric field is radial and has uniform magnitude over the rounded portion of the Gaussian surface. The contribution of that portion to the integral on the left side of Gauss' law is $2\pi rLE$. The field is parallel to the cylinder ends (it is perpendicular to $\mathrm{d}\vec{A}$), so the ends do not contribute to the integral. Thus

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = 2\pi rLE \,.$$

In part (a) the charge enclosed by the Gaussian surface is all the charge in a length L of the tube. This is λL. Substitute $q_{\mathrm{enc}} = \lambda L$ into Gauss' law and solve for E.

In part (b) the Gaussian cylinder is entirely inside the tube, so the charge enclosed is zero. Substitute this value into Gauss' law and solve for E.

Chapter 24 Hint for Problem 20

Use Gauss' law:

$$\epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{A} = q_{\text{enc}} \,,$$

where the integral is over a Gaussian surface and q_{enc} is the charge enclosed by the Gaussian surface.

Take the Gaussian surface to a cylinder of radius r and length L, concentric with the wire and cylinder. Since you want the electric field at a point outside the cylinder, take r to be greater than the cylinder radius.

The electric field, if any, must be radial. This means the field is parallel to $\mathrm{d}\vec{A}$ on the rounded portion of the Gaussian surface and is perpendicular to $\mathrm{d}\vec{A}$ on the ends. Furthermore, the magnitude of the field is uniform over the rounded portion of the Gaussian surface. Thus

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = 2\pi r L E \,.$$

Let λ be the linear charge density of the wire and σ be the area charge density of the cylinder. Then the charge enclosed by the Gaussian surface is

$$q_{\text{enc}} = \lambda L + 2\pi R L \sigma \,,$$

where R is the radius of the cylinder. Substitute this expression into Gauss' law, set E equal to zero, and solve for σ.

Use Gauss' law:

$$\epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{A} = q_{\text{enc}} \,,$$

where the integral is over a Gaussian surface and q_{enc} is the net charge enclosed by the surface. For both parts take the Gaussian surface to be a cylinder of radius r and length L that is concentric with the cylinder of charge. The electric field is radially outward and has uniform magnitude over the rounded portion of the Gaussian surface. The ends of the surface do not contribute to the integral on the left side of Gauss' law and the rounded portion contributes $2\pi rLE$. Thus

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = 2\pi rLE \,.$$

In part (a) the radius of the Gaussian surface is $r = 3.0\,\text{cm}$ and is less than the radius of the cylinder of charge. The charge enclosed by the Gaussian surface is given by the integral over the volume of the cylinder of charge that is within the Gaussian surface:

$$q_{\text{enc}} = \int \rho \,\mathrm{d}V \,.$$

This integral can be evaluated by taking the infinitesimal volume element $\mathrm{d}V$ to be a cylindrical shell of radius r' and thickness $\mathrm{d}r'$. The volume is $\mathrm{d}V = 2\pi r'L\,\mathrm{d}r'$. When $A(r')^2$ is substituted for ρ the integral becomes

$$q_{\text{enc}} = \int_0^r 2\pi r'LA(r')^2 \,\mathrm{d}r' \,.$$

Evaluate the integral and substitute the result into the right side of Gauss' law. Solve for E.

In part (b) the Gaussian surface is outside the cylinder of charge and the charge it encloses is given by

$$q_{\text{enc}} = \int_0^R \rho \,\mathrm{d}V = \int_0^R 2\pi r'LA(r')^2 \,\mathrm{d}r' \,.$$

Evaluate this integral, substitute the result into the right side of Gauss' law, and solve for E.

Chapter 24 Hint for Exercise 26

The magnitude of the electric field due to a large uniform sheet of charge is given by $E = \sigma/2\epsilon_0$, where σ is the area charge density. For each region, vectorially add the individual fields due to the sheets. In (a) both fields point upward, in (b) one field points upward and other points downward, and in (c) both fields point downward.

Chapter 24 Hint for Exercise 28

Let $\vec{E}_u$ be the electric field of a large uniform sheet with area charge density σ, $\vec{E}_d$ be the field of a uniform disk with the same charge density, and $\vec{E}$ be the field of a sheet with a hole. Then, $\vec{E} = \vec{E}_u - \vec{E}_d$. $\vec{E}_u$ has magnitude $\sigma/2\epsilon_0$ and points away from the sheet if σ is positive. According to Eq. 24–27, $\vec{E}_d$ has magnitude $(\sigma/2\epsilon_0)[1 - z/\sqrt{z^2 + R^2}]$ and points away from the disk.

Chapter 24 Hint for Problem 32

Each plate produces a field of magnitude $\sigma/2\epsilon_0$, where σ is the area charge density on the plate. In the region between the plates, the two fields are in the same direction so the magnitude of the total field there is given by σ/ϵ_0. Solve for σ. The magnitude of the charge on either plate is $q = \sigma A$, where A is the area of the plate.

(a) Remember that the total flux through any surface is equal to $q_{\text{enc}}/\epsilon_0$, where q_{enc} is the charge enclosed by the surface. The total flux changes if and only if the net charge enclosed changes.

(b) Use gauss' law in the form

$$\epsilon_0 \Phi = q_{\text{enc}} \, .$$

(a) The charge is given by the volume integral

$$Q = \int \rho \, \mathrm{d}V$$

over the sphere. Since the charge density has spherical symmetry (it depends only on r), the infinitesimal volume may be replaced by the volume of a spherical shell with infinitesimal thickness $\mathrm{d}r$: $\mathrm{d}V = 4\pi r^2 \, \mathrm{d}r$. Thus

$$Q = \frac{4\pi\rho_s}{R} \int_0^R r^3 \, \mathrm{d}r \, .$$

Evaluate the integral.

(b) Draw a diagram that shows a spherical Gaussian surface with radius r, inside the sphere of charge. The electric field is radial, so

$$\int \vec{E} \cdot \mathrm{d}\vec{A} = 4\pi r^2 E \, .$$

The charge enclosed is

$$q_{\text{enc}} = \frac{4\pi\rho_s}{R} \int_0^r r^3 \, \mathrm{d}r \, .$$

Evaluate the integral, then substitute into Gauss' law and solve for E. Replace ρ_s with $Q/\pi R^3$ to obtain the result requested.

(a) Draw a diagram with a Gaussian surface in the form a sphere with radius r, inside the charge distribution. The field is radial, so

$$\int \vec{E} \cdot \mathrm{d}\vec{A} = 4\pi r^2 E\,.$$

Since the volume of Gaussian sphere is $4\pi r^3/3$ and the charge is uniformly distributed, the charge enclosed is

$$q = \frac{4\pi\rho r^3}{3}\,.$$

Use Gauss' law to show that

$$E = \frac{\rho r}{3\epsilon_0}\,.$$

The field is in the direction of $\rho\vec{r}$, radially outward if ρ is positive and radially inward if ρ is negative.

(b) Let $\vec{E}_s$ be the field of the solid sphere, $\vec{E}_c$ be the field of a sphere that just fills the cavity and has the same volume charge density as the actual sphere, and $\vec{E}$ be the field of the sphere with the cavity. Then, $\vec{E} = \vec{E}_s - \vec{E}_c$. Substitute

$$\vec{E}_s = \frac{\rho\vec{r}}{3\epsilon_0}\,.$$

Since $\vec{r} - \vec{a}$ is the vector from the center of the cavity to the point P,

$$\vec{E}_c = \frac{\rho(\vec{r} - \vec{a})}{3\epsilon_0}\,.$$

Also substitute this expression.