(a) Use F = eE, where F is the magnitude of the force on the proton and E is the magnitude of the electric field at the position of the proton. A proton is positively charged so the force on it is in the direction of the field.

(b) Assume that the separation along a line that is perpendicular to the page at B is the same as the separation along a line that is perpendicular to the page at A so that half as many lines pass through a given area perpendicular to the page at B as pass through the same area at A. Then, the field is half as strong at B as at A.

[ans: (a) 6.4×10^{-18} N, to the left; (b) 20 N/C]

Solve

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$$

for |q|. Here E is the magnitude of the electric field created by point charge q at a point that is r distant from the charge.

The fields due the two 5.0q charges have the same magnitudes at P but point in opposite directions, so they sum to 0. The field due to the 3.0q charge has magnitude

$$E_{3q} = \frac{1}{4\pi\epsilon_0} \frac{3.0q}{d^2}$$

and points away from the charge. The field due to the -12q charge has magnitude

$$E_{12q} = \frac{1}{4\pi\epsilon_0} \frac{12q}{(2d)^2}$$

and points toward the charge. Sum these fields vectorially.

 $\begin{bmatrix} ans: & 0 \end{bmatrix}$

Draw a diagram, similar to Fig. 23–31, and show the electric field vector for the field produced at P by each charge. Each charge is a distance $\sqrt{r^2 + d^2/4}$ from P. The components of the two fields along the perpendicular bisector sum to zero and the components parallel to the line joining the charges sum to

$$E = \frac{qd}{4\pi\epsilon_0} \frac{1}{(r^2 + d^2/4)^{3/2}} \,.$$

For points that are far from the dipole, $r \gg d/2$ and $(r^2 + d^2/4)^{-3/2} \approx 1/r^3$. Substitute the magnitude p of the dipole moment for qd. The dipole moment is directed from the negative charge toward the positive charge and the direction of the field is opposite that of the dipole moment.

The diagram shows the upper half of the semicircle. The x component of the electric field is given by

$$E_x = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\sin\theta}{r^2} \,\mathrm{d}s\,,$$

where ds is a line element of the semicircle and λ is the linear charge density. Since charge q is distributed along a line of length $\pi r/2$, $\lambda = 2q/\pi r$. Change the variable of integration to θ by making the substitution ds = $r d\theta$, integrate from $\theta = 0$ to $\theta = 90^{\circ}$. You should get

$$E_x = \frac{q}{2\pi^2 \epsilon_0 r^2}$$



The y component of the electric field is given by

$$E_y = -\frac{1}{4\pi\epsilon_0} \int \frac{\lambda\cos\theta}{r^2} \,\mathrm{d}s \,.$$

Make the same substitutions and obtain $E_y = -q/2\pi^2 r^2$. Now, carry out a similar derivation for the bottom half of the semicircle. You should obtain

$$E_x = -\frac{q}{2\pi^2 \epsilon_0 r^2}$$

and

$$E_y = -\frac{q}{2\pi^2\epsilon_0 r^2} \,.$$

The total field at P is the vector sum of these two fields.

 $\begin{bmatrix} ans: Q/\pi^2 \epsilon_0 r^2, downward \end{bmatrix}$

Place the x axis along the rod with the origin at the midpoint of the rod, as shown. Consider an infinitesimal segment of the rod located at x and with width dx. It contains charge $dq = \lambda dx$, where λ is the linear charge density of the rod. The electric field produced at point P by the segment has magnitude

$$\mathrm{d}E = \frac{1}{4\pi\epsilon_0} \,\frac{\lambda\,\mathrm{d}x}{r^2}\,,$$



where r is the distance from the segment to P. The direction of the field is shown on the diagram.

Make a symmetry argument to show that the horizontal component of the total field at P is zero. The vertical component is given by the integral

$$E_v = \frac{1}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{\lambda \cos\theta}{r^2} \,\mathrm{d}x \,.$$

Both θ and r depend on x. Replace r with $\sqrt{x^2 + y^2}$ and $\cos \theta$ with $y/\sqrt{x^2 + y^2}$, then evaluate the integral. You may need to know that

$$\int \frac{\mathrm{d}x}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

This integral can be found in the Appendix E of the text.

Evaluate Eq. 23–6 for the magnitude E of the electric field at a point on the central axis, a distance z from the disk center:

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \,,$$

where R is the radius of the disk and σ is the area charge density on the disk.

The magnitude of the force of gravity on the proton is given by $F_g = mg$, where *m* is the mass of the proton and *g* is the free-fall acceleration. The force of an electric field *E* has magnitude $F_E = eE$, where *e* is the charge on the proton $(1.60 \times 10^{-19} \text{ C})$. You want these two forces to have equal magnitudes.

The force of gravity is downward, so the electric force must be upward. since the charge on the proton is positive the force is in the same direction as the field.

(a) and (b) If the electron is originally traveling in the positive direction, its acceleration is a = -eE/m, where E is the magnitude of the field and m is the mass of the electron. The kinematic equations are

$$x = v_0 t - \frac{1}{2} (eE/m)t^2$$

and

$$v = v_0 - (eE/m)t,$$

where v_0 is its initial velocity. Set v = 0 and solve for x and t.

(c) If ℓ is the width of the region, then the work done by the electric field is $W = -eE\ell$. According to the work-kinetic energy theorem the magnitude of W is the kinetic energy lost by the electron. Calculate $|W|/K_0$, where $K_0 = \frac{1}{2}mv_0^2$ is the initial kinetic energy.

Since the field is uniform, the acceleration of the electron is constant. Its magnitude is given by a = eE/m, where E is the magnitude of the electric field and m is the mass of the electron. If d is the separation of the plates, the kinematic equations are

$$d = \frac{1}{2}(eE/m)t^2$$

and

$$v = (eE/m)t\,,$$

where t is the time of flight. The origin was placed at the initial position of the electron. Solve these equations simultaneously for v and E.

[ans: (a) $2.7 \times 10^6 \,\mathrm{m/s}$; (b) $1000 \,\mathrm{N/C}$]

The torque is given by $\vec{\tau} = \vec{p} \times \vec{E}$ and its magnitude is given by $\tau = pE \sin \theta$, where \vec{p} is the dipole moment, \vec{E} is the electric field, and θ is the angle between them when the vectors are drawn with their tails at the same point. The magnitude of the dipole moment is 2ed, where d is the separation of the charges. In (a) $\theta = 0^{\circ}$, in (b) $\theta = 90^{\circ}$, and in (c) $\theta = 180^{\circ}$.

[ans: (a) 0; (b) $8.5 \times 10^{-22} \,\mathrm{N \cdot m}$; (c) 0]

The work required of an external agent is the change in the potential energy of the dipole. The initial potential energy is $U_i = -pE\cos\theta_0$; the final potential energy is $U_f = -pE\cos(\theta_0 + 180^\circ)$. Use the trigonometric identity $\cos(\theta_0 + 180^\circ) = -\cos\theta_0$