

CHAPTER 21      HINT FOR EXERCISE 2

The change in entropy is given by

$$\Delta S = \int \frac{dQ}{T},$$

where  $T$  is the temperature in kelvins. Since the process is isothermal  $\Delta S = Q/T$ , where  $Q$  is the energy taken in as heat. Solve for  $Q$ . Use  $T = T_C + 273$  to convert the temperature  $T_C$  in degrees Celsius to kelvins.

(a) In an isothermal expansion the temperature has the same value at every stage.

(b) In a constant pressure process the pressure has the same value at every stage. Since the gas is ideal this means the change in temperature is

$$\Delta T = \frac{p \Delta V}{nR}.$$

Since  $\Delta V = V_0$ ,  $\Delta T = pV_0/nR = T_0$  and the final temperature is  $2T_0$ .

(c) For an adiabatic process

$$p_0 V_0^\gamma = p V^\gamma,$$

where  $\gamma$  is the ratio  $C_p/C_V$  of the heat capacities. Since the gas is ideal

$$T_0 V_0^{\gamma-1} = T V^{\gamma-1}.$$

Thus

$$T = T_0 \left( \frac{V_0}{V} \right)^{\gamma-1}.$$

Since the gas is monatomic  $\gamma = 1.667$ . Use  $V_0/V = 1/2$ .

(d) The change in entropy is given by

$$\Delta S = \int \frac{dQ}{T}$$

and for any infinitesimal part of a process

$$dQ = dE_{\text{int}} + dW = nC_V dT + p dV.$$

For each of the three processes  $dV$  is positive. For the isothermal process  $dT$  is zero and for the isobaric process  $dT$  is positive. Use this to find the signs of  $dQ$  and  $\Delta S$ . The temperature decreases during the adiabatic process but you immediately know something about the energy absorbed as heat during this process.

## CHAPTER 21      HINT FOR PROBLEM 8

(a) During any infinitesimal portion of the process the energy  $dQ$  taken in by the gas as heat is related to the change  $dS$  in the entropy of the gas by

$$dQ = T dS ,$$

where  $T$  is the temperature of the gas. Thus for the whole process

$$Q = \int T dS .$$

This is the area under the graph.

(b) For any process the change  $\Delta E_{\text{int}}$  in the internal energy of the gas is related to the change  $\Delta T$  in the temperature by

$$\Delta E_{\text{int}} = nC_V \Delta T ,$$

where  $C_V$  is molar heat capacity. In this case the gas is monatomic, so  $C_V = (3/2)R$ .

(c) Use the first law of thermodynamics:  $\Delta E_{\text{int}} = Q - W$ .

CHAPTER 21 HINT FOR PROBLEM 12

(a) Points 1 and 2 lie on the same isothermal so, according to the ideal gas law,

$$p_1 V_1 = p_2 V_2 .$$

Substitute  $V_2 = 3.00V_1$  and solve for  $p_2$ .

Points 1 and 3 lie on the same adiabat so

$$p_1 V_1^\gamma = p_3 V_3^\gamma .$$

Substitute  $V_3 = 3.00V_1$  and solve for  $p_3$ . For a diatomic gas  $\gamma = 1.4$ .

To find  $T_3$ , use the ideal gas law in the form

$$\frac{p_1 V_1}{T_1} = \frac{p_3 V_3}{T_3} .$$

(b) Over the isothermal portion of the cycle, the work done by the gas is

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{nRT_1}{V} dV = nRT_1 \ln \frac{V_2}{V_1} .$$

Substitute for  $V_2$ .

The change in internal energy is zero since, for an ideal gas, it depends only on the temperature. The first law of thermodynamics then reduces to  $Q = W$ . The change in entropy is given by

$$\Delta S = \frac{Q}{T_1} .$$

Over the constant volume portion of the cycle  $W = 0$ . The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V \Delta T = (5/2)nR(T_3 - T_2) .$$

Substitute for the temperatures in terms of  $R$  and  $T_1$ . The first law reduces to

$$Q = \Delta E_{\text{int}} .$$

The change in entropy is given by

$$\Delta S = \int \frac{dQ}{T} = nC_V \int_{T_2}^{T_3} \frac{dT}{T} = (5/2)nR \ln(T_3/T_2) .$$

Substitute for the temperatures.

Over the adiabatic portion of the cycle  $Q = 0$ .

The work is given by

$$W = \int_{V_3}^{V_1} p dV = \int_{V_3}^{V_1} \frac{p_1 V_1^\gamma}{V^\gamma} dV = \frac{p_1 V_1^\gamma}{1 - \gamma} [V_1^{1-\gamma} - V_3^{1-\gamma}] .$$

Substitute for  $V_3$ .

The first law reduces to  $\Delta E_{\text{int}} = -W$ .

The change in entropy is 0.

## CHAPTER 21      HINT FOR PROBLEM 13

(a) The heat absorbed by the lead is given by  $m_L c_L (T_F - T_L)$ , where  $m_L$  is the mass of the lead,  $c_L$  is its specific heat (128 J/kg · K),  $T_L$  is its initial temperature, and  $T_F$  is the final temperature. The heat rejected by the copper is  $m_C c_C (T_F - T_C)$ , where  $m_C$  is the mass of the copper,  $c_C$  is its specific heat (386 J/kg · K), and  $T_C$  is its initial temperature. Since no energy entered or left the two-block system, these heats must sum to zero. Solve

$$m_L c_L (T_F - T_L) + m_C c_C (T_F - T_C) = 0$$

for  $T_F$ .

(b) Since no work is done, the energy rejected as heat by the copper comes from its store of internal energy, while the energy accepted as heat by the lead enters its store of internal energy. No energy is lost so these are equal in magnitude.

(c) The change in the entropy of the copper is

$$\Delta S_C = \int \frac{dQ}{T} = m_C c_C \int_{T_i}^{T_f} \frac{dT}{T} = m_C c_C \ln \frac{T_F}{T_C}.$$

Similarly, the change in the entropy of the lead is

$$\Delta S_L = m_L c_L \ln \frac{T_F}{T_L},$$

Add the two values to obtain the change in the entropy of the system.

[ ans:      (a) 320 K; (b) 0; (c) 1.72 J/K ]

CHAPTER 21      HINT FOR PROBLEM 15

Mass  $m$  of ice changes temperature from  $T_1$  ( $= -10^\circ\text{C}$ ) to  $T_2$  ( $= 0^\circ\text{C}$ ). It absorbs energy  $Q_1 = mc_i(T_2 - T_1)$  as heat and its entropy changes by

$$\Delta S_1 = \int \frac{dQ}{T} = mc_i \int_{T_1}^{T_2} \frac{dT}{T} = mc_i \ln \frac{T_2}{T_1}.$$

Here  $c_i$  is the specific heat of ice (2220 J/kg). The ice then melts. It absorbs energy  $Q_2 = mL_F$  as heat. Here  $L_F$  is the heat of fusion for water ( $333 \times 10^3$  J/kg). Its entropy increases by

$$\Delta S_2 = \frac{Q_2}{T_2}.$$

Finally, the temperature of the resulting water increases from  $T_2$  to the final equilibrium temperature  $T_3$  ( $= 15^\circ\text{C}$ ). It absorbs energy  $Q_3 = mc_w(T_3 - T_2)$  as heat. Here  $c_w$  is the specific heat of water (4190 J/kg). Its entropy increases by

$$\Delta S_3 = m_i c_w \ln \frac{T_3}{T_2}.$$

Since the mass of water in the lake is so large its temperature does not change. Its entropy changes by

$$\Delta S_L = -\frac{Q_1 + Q_2 + Q_3}{T_3}.$$

Don't forget to convert  $T_3$  to kelvins when computing  $\Delta S_2$  and  $\Delta S_L$ .

[ans:    0.76 J/K]

## CHAPTER 21      HINT FOR PROBLEM 16

First find the final temperature of the system. Let  $m_I$  be the mass of the ice cube,  $m_W$  be the mass of the original water,  $T_I$  be the initial temperature of the ice, and  $T_W$  be the initial temperature of the water. Assume that all the ice melts and the temperature of the resulting ice water increases to  $T_f$ . The energy absorbed as heat by the ice and the resulting ice water is given by

$$Q = m_I c_I (T_m - T_I) + m_I L_I + m_I c_W (T_f - T_m),$$

where  $T_m$  is the melting point of ice ( $0^\circ C$  or  $273 K$ ),  $c_I$  ( $= 2220 \text{ J/kg} \cdot K$ ) is the specific heat of ice,  $c_W$  ( $= 4190 \text{ J/kg} \cdot K$ ) is the specific heat of water, and  $L_I$  ( $= 333 \times 10^3 \text{ J/kg}$ ) is heat of fusion of water.

The temperature of the original water decreases from  $T_W$  to  $T_f$  and the energy removed from it as heat is given by

$$Q = m_W c_W (T_W - T_f).$$

Since all of the energy removed from the original water goes into the ice and the resulting ice water, these two expressions for  $Q$  must be equal. Equate the two expressions to each other and solve for  $T_f$ . You may use values of  $T_I$ ,  $T_W$ , and  $T_m$  on either the Celsius or Kelvin scale. The answer will be on the same scale.

Now compute the entropy changes.

The entropy change for the ice as its temperature increases from  $T_I$  to  $T_m$  is

$$\Delta S_1 = \int_{T_I}^{T_m} \frac{m_I c_I}{T} dT = m_I c_I \ln \frac{T_m}{T_I}.$$

The entropy change for the ice as it melts is

$$\Delta S_2 = \frac{m_I L_I}{T_m}.$$

The entropy change for the ice water as its temperature increases from  $T_m$  to  $T_f$  is

$$\Delta S_3 = \int_{T_m}^{T_f} \frac{m_I c_W}{T} dT = m_I c_W \ln \frac{T_f}{T_m}.$$

The entropy change for the original water as its temperature decreases from  $T_W$  to  $T_f$  is

$$\Delta S_4 = \int_{T_W}^{T_f} \frac{m_W c_W}{T} dT = m_W c_W \ln \frac{T_f}{T_W}.$$

You must use temperatures on the Kelvin scale to evaluate these expressions.

To find the entropy change for the system, add the four contributions.

(a) Evaluate

$$p = (5.00 \text{ kPa})e^{(V_i - V)/a}$$

with  $V = V_f$ .

(b) Solve the ideal gas equation

$$p_f V_f = nRT_f$$

for  $T_f$ . Here  $p_f$  is the final pressure and  $T_f$  is the final temperature.

(c) The work done by the gas is given by

$$W = \int_{V_i}^{V_f} p \, dV = (5.00 \times 10^3 \text{ Pa}) \int_{V_i}^{V_f} e^{(V_i - V)/a} \, dV.$$

To evaluate the integral note that  $e^{(V_i - V)/a} = e^{V_i/a} e^{-V/a}$ . Thus

$$W = (5.00 \times 10^3 \text{ Pa}) e^{V_i/a} \int_{V_i}^{V_f} e^{-V/a} \, dV.$$

(d) Consider an isothermal expansion from volume  $V_i$  to volume  $V_f$  at temperature  $T_i$ , followed by a change in temperature from  $T_i$  to  $T_f$  at constant volume ( $V_f$ ). For the first process the internal energy of the gas does not change, so according to the first law of thermodynamics the energy taken in as heat is equal to the work done by the gas. The entropy change is

$$\Delta S_1 = \frac{Q}{T_i} = \frac{W}{T_i}.$$

The work is

$$W = \int_{V_i}^{V_f} p \, dV = \int_{V_i}^{V_f} \frac{nRT_i}{V} \, dV,$$

where the ideal gas law was used to substitute for  $p$ . Note that this process is quite different from the actual process.

For the second process

$$\Delta S_2 = \int_{T_i}^{T_f} \frac{nC_V}{T} \, dT,$$

where  $C_V (= \frac{3}{2}RT)$  is the molar specific heat at constant volume for a monatomic ideal gas.

The entropy change for the gas is the sum of the entropy changes for the two processes.



## CHAPTER 21      HINT FOR EXERCISE 24

The efficiency of a Carnot engine is given by

$$e = 1 - \frac{T_L}{T_H},$$

where  $T_L$  is the temperature of the low-temperature reservoir and  $T_H$  is the temperature of the high-temperature reservoir. These temperatures must be in kelvins. For each of the two efficiencies solve for  $T_H$ , then calculate the difference.

During any infinitesimal portion of the cycle the energy taken in by the working substance as heat is given by  $dQ = T dS$ , where  $T$  is the temperature and  $dS$  is the entropy change. During any finite portion

$$Q = \int T dS.$$

During portions in which the entropy is increasing  $Q$  is positive, indicating that energy is entering the working substance. During portion in which the entropy is decreasing  $Q$  is negative, indicating that energy is leaving the working substance. Over the whole cycle  $Q = \int T dS$  gives the net energy transferred to the working substance.

For each portion the integral can be computed as the area under the curve. Thus its value is the area under that portion of the curve for which the entropy is increasing minus the area under that portion for which the entropy is decreasing.

Use the definition of efficiency for a heat engine. The total work done is  $W_1 + W_2$ , where  $W_1$  is the work done by the first stage and  $W_2$  is the work done by the second stage. The energy input is the energy  $Q_1$  taken in as heat at the high temperature  $T_1$ . Thus

$$\varepsilon = \frac{W_1 + W_2}{Q_1}.$$

The first law of thermodynamics tells us that  $W_1 = Q_1 - Q_2$  and  $W_2 = Q_2 - Q_3$ . Make these replacements in the definition of efficiency and divide both the numerator and denominator by  $Q_1$ . Since both stages are ideal (reversible)

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

and

$$\frac{Q_2}{Q_3} = \frac{T_2}{T_3}.$$

Thus

$$\frac{Q_3}{Q_1} = \frac{T_3}{T_1}.$$

Make this substitution.

CHAPTER 21      HINT FOR PROBLEM 32

(a) For the adiabatic process BC,

$$p_B V_B^\gamma = p_C V_C^\gamma,$$

where  $\gamma$  is the ratio  $C_p/C_V$  of the heat capacities. Use  $p_C = p_B/32$  and  $V_C = 8V_B$ , then solve for  $\gamma$ . If the gas is monatomic  $\gamma = 5/3$ ; if it is diatomic  $\gamma = 7/5$ ; if it is polyatomic  $\gamma = 4/3$ .

(b) The efficiency is the net work done by the engine divided by the energy taken in as heat. Energy is taken in as heat only during the AB portion of the cycle. (Energy is released as heat during the CD portion and no energy is transferred as heat during the adiabatic portions.) The portion AB is at constant pressure, so energy taken in as heat is given by  $Q_{AB} = nC_p \Delta T$ , where  $C_p$  is the molar heat capacity at constant pressure. Use the ideal gas law  $pV = nRT$  to write  $\Delta T$  in terms of the pressures and volumes. Also use  $C_p = (5/2)R$ , the value of the molar heat capacity for a monatomic gas. You should obtain  $Q_{AB} = (5/2)p_0V_0$ .

You must now find expressions in terms of  $p_0$  and  $V_0$  for the work done during each part of the cycle. Use

$$W = \int p \, dV.$$

For each of the two constant pressure processes  $W = p \Delta V$ . For each of the two adiabatic processes  $pV^\gamma = K$ , where  $K$  is a constant. Thus  $p = KV^{-\gamma}$  and the work done during the BC portion of the cycle is

$$W_{BC} = \int KV^{-\gamma} \, dV = \frac{K}{1-\gamma} V^{1-\gamma} \Big|_B^C = \frac{pV}{1-\gamma} \Big|_B^C = \frac{p_C V_C - p_B V_B}{1-\gamma}.$$

You should get  $W_{BC} = 2.25p_0V_0$ . Similarly,  $W_{DA} = -1.125p_0V_0$ .

## CHAPTER 21      HINT FOR EXERCISE 34

(a) The coefficient of performance  $K$  of a refrigerator is given by

$$K = \frac{|Q_L|}{|W|},$$

where  $Q_L$  is the energy absorbed as heat by the working substance from the cold compartment and  $W$  is the work done by the working substance, both during one cycle. According to the problem statement  $Q_L = 600 \text{ J}$  and  $W = -200 \text{ J}$ .

(b) The first law of thermodynamics tells us that

$$\Delta E_{\text{int}} = Q_L + Q_H - W.$$

The magnitude of  $Q_H$  gives the energy that is transferred to the high temperature reservoir (the kitchen).  $Q_H$  is actually negative since energy leaves the working substance. Over a cycle the change  $\Delta E_{\text{int}}$  in the internal energy of the working substance is zero. Solve for  $|Q_H|$ .

## CHAPTER 21      HINT FOR PROBLEM 40

The energy that is extracted as heat from the freezing compartment is given by  $Q_L = KW$ , where  $K$  is coefficient of performance and  $W$  is the work done by the refrigerator on the working substance. The coefficient of performance of an ideal refrigerator is given by

$$K = \frac{T_H - T_L}{T_C},$$

where  $T_H$  is the temperature of the outside air and  $T_C$  is the temperature of the freezing compartment, both in kelvins. The work done in time  $\Delta t$  is  $W = P \Delta t$ , where  $P$  is the power of the refrigerator.

## CHAPTER 21      HINT FOR PROBLEM 44

The total number of microstates for a configuration in which  $n_1$  molecules are on one side of the box and  $n_2$  molecules are on the other side is

$$W = \frac{N!}{n_1!n_2!}.$$

Here  $n_1 = N/2$  and  $n_2 = N/2$ .  $N$  is the total number of molecules (50 for parts (a), (b), and (c), 100 for part (d), and 200 for part (e)).

The total number of microstates is given by  $N_{\text{micro}} = 2^N$  (see Exercise 45).

The fraction of the time that the system spends with half the molecules on each side of the box is

$$f = \frac{W}{N_{\text{micro}}}.$$

As  $N$  increases, which increases more rapidly,  $W$  or  $N_{\text{micro}}$ ?