CHAPTER 2 HINT FOR PROBLEM 5

Let d be the total distance and Δt be the time for either of the one-way trips. The trip is composed of two parts, with different speeds: $d = d_1 + d_2$ and $\Delta t = \Delta t_1 + \Delta t_2$. The partial displacements and times are related by $d_1 = v_1 \Delta t_1$ and $d_2 = v_2 \Delta t_2$.

For (a), $\Delta t_1 = \Delta t_2$ so $d = (v_1 + v_2)\Delta t_1$. Divide by $\Delta t = \Delta t_1 + \Delta t_2 = 2\Delta t_1$.

For (b), $d_1 = d_2$, so $d = 2d_1$ and

$$\Delta t = \left(\frac{1}{v_1} + \frac{1}{v_2}\right) d_1 \,.$$

To compute the average speed, divide the expression for d by the expression for Δt.

(c) To find the average speed over the round trip, divide the total distance $2d$ by the total time. Let $s_{\text{avg } t}$ be the average speed to Houston and $s_{\text{avg } f}$ be the average speed from Houston. Then, the time for the first one-way trip is

$$\Delta t_t = \frac{d}{s_{\text{avg } t}} \,,$$

the time for the second one-way trip is

$$t_f = \frac{d}{s_{\text{avg } f}} \,,$$

and the total time is

$$t = \left(\frac{1}{s_{\text{avg } t}} + \frac{1}{s_{\text{avg } f}}\right) d \,.$$

(d) Now you must compute the average velocity, not the average speed. This means you must divide the displacement for the round trip by the time for that trip. Note that you begin and end the trip at the same place.

(e) Your graph will consist of two straight line segments with different slopes. When you have drawn them, draw the line with a slope that represents the average velocity.

[ans: (a) 73 km/h; (b) 68 km/h; (c) 70 km/h; (d) 0]

Chapter 2 Hint for Problem 8

First calculate the time until the trains meet. This is their original separation divided by the sum of their speeds. Then calculate the distance the bird flys in this time. It is the product of the bird's speed and the time.

CHAPTER 2 HINT FOR EXERCISE 11

(a) Differentiate $x(t)$ with respect to t to obtain an expression for the velocity at any time. Substitute $t = 1\,\mathrm{s}$ into this expression.

(b) If v is positive the particle is moving in the positive x direction, if v is negative it is moving in the negative x direction.

(c) The speed is the magnitude of the velocity.

(d) Look at the expression for the velocity as a function of time and ask what happens as t increases from $1\,\mathrm{s}$. You will find several answers: over the short term the speed is doing one thing; at longer times it is doing another.

(e) Look at the expression for the velocity as a function of time and notice that the initial velocity is negative but that the velocity is increasing algebraically (the acceleration is positive).

(f) Notice that the velocity is zero at $t = 2\,\mathrm{s}$ and then continues to increase without change in sign.

[ans: (a) $-6\,\mathrm{m/s}$; (b) negative x direction; (c) $6\,\mathrm{m/s}$; (d) speed decreases until $t = 2\,\mathrm{s}$, then increases; (e) yes; (f) no]

(a) The average velocity is given by

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}\,,$$

where x_i is the initial coordinate ($x(t)$ evaluated for $t = 2.00\,\text{s}$) and x_f is the final coordinate ($x(t)$ evaluated for $t = 3.00\,\text{s}$).

(b), (c), and (d) Differentiate $x(t)$ with respect to t to find an expression for the velocity as a function of time. Evaluate the expression for the three given times.

(e) First you must find the coordinate of the midpoint. It is given by

$$x_m = \frac{x_f + x_i}{2}\,.$$

Now, find the time the particle is at the midpoint: solve $x_m = 9.75 + 1.50t^3$ for t. This is a cubic equation. Look up a technique for solving it or else use a trial and error method. Systematically substitute various values of t into $9.75 + 1.50t^3$ until you get an answer that is close to x_m. Keep a record of the results to help you decide on your next trial value. Once you have found a value for t substitute it into your expression for the velocity.

Notice that all these velocities are different. The average velocity is not the same as the velocity at the midpoint in position nor is it the same as the velocity at the midpoint in time. It also is not the average of the velocities at the end points. If the acceleration were constant the average velocity over the interval, the average of the initial and final velocities, and the instantaneous velocity at the midpoint in time, would all be the same and they would be different from the instantaneous velocity at the midpoint in position.

(f) To construct the graph draw the appropriate lines, with slopes equal to the various average and instantaneous velocities requested.

(a) The average velocity over an interval of time is given by

$$v_{\text{avg}} = \frac{x_f - x_i}{\Delta t}\,,$$

where x_f is the coordinate at the end of the interval ($t = 8.00\,\text{min}$), x_i is the coordinate at the beginning ($t = 2.00\,\text{min}$), and Δt is the duration of the interval. You may place the origin of the x axis at the initial position of the man. Then $x_i = 0$. At time $t = 8.00\,\text{min}$ the man has walked for $3.00\,\text{min}$ at $2.20\,\text{m/s}$. Use this information to calculate x_f. You must convert $3.00\,\text{min}$ to seconds.

(b) The average acceleration over an interval of time is given by

$$a_{\text{avg}} = \frac{v_f - v_i}{\Delta t}\,,$$

where v_f is the velocity at the end of the interval and v_i is the velocity at the beginning. At $t = 2.00\,\text{min}$ the man is standing still and at $t = 8.00\,\text{min}$ he is moving at $2.20\,\text{m/s}$.

(c) and (d) Repeat the calculations for the new time interval.

(e) The graphs all consist of straight line segments. Remember that the average velocity over an interval is the slope of a certain line on a graph of the coordinate as a function of the time and the average acceleration over an interval is the slope of a certain line on a graph of the velocity as a function of the time.

Chapter 2 Hint for Problem 20

First find the time at which the electron stops. Its velocity as a function of time is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[16te^{-t}\right] = [16 - 16t]\, e^{-t}$$

in meters per second. Set $v = 0$ and solve for t. Then evaluate $x = 16te^{-t}$ m for that value of t.

Chapter 2 Hint for Exercise 26

Put the x axis along the barrel, with the origin at the point where the bullet is fired. Take the time to be zero when the bullet is fired. Since the bullet starts from rest its coordinate is given by

$$x = \frac{1}{2}at^2$$

and its velocity is given by

$$v = at\,,$$

where a is its acceleration. Solve these equations simultaneously for t. The other unknown quantity is the acceleration a. Use one of the equations to eliminate it from the other.

The second equation gives

$$a = \frac{v}{t}\,.$$

When v/t is substituted for a in the first equation the result is

$$x = \frac{1}{2}vt\,.$$

Solve for t.

Chapter 2 Hint for Exercise 28

Take the time to be zero at the instant the jet plane starts its run and place the origin of the coordinate system at the point where it starts its run. At that instant its velocity is zero. Take the time to be t when the plane takes off at the end of the runway. Then its coordinate is $x = 1.80\,\mathrm{km}$ and its velocity is $v = 360\,\mathrm{km/h}$. Convert the coordinates and velocities to standard SI units.

The coordinate of the plane is given by

$$x = \frac{1}{2}at^2$$

and the velocity is given by

$$v = at\,.$$

The equations were obtained by setting $x_0 = 0$ and $v_0 = 0$ in the usual kinematic equations. The two unknown quantities are the acceleration a of the plane and the time t of takeoff. Use the second equation to eliminate the unknown t from the first. That is, find an expression for t in terms of a and substitute it for t in the first equation. You should get

$$x = \frac{v^2}{2a}\,.$$

Now, solve for a and substitute numerical values.

As an alternative, you might solve the kinematic equation

$$v^2 = v_0^2 + 2a(x - x_0)$$

for a. Here t has already been eliminated for you.

Chapter 2 Hint for Exercise 32

The coordinate of the particle is given by

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \,,$$

where x_0 is its coordinate and v_0 is its velocity at time $t = 0$. Its acceleration is denoted by a.

According to the graph $x = -2\,\text{m}$ at $t = 0$, so $x_0 = -2\,\text{m}$.

The graph shows that $x = 0$ at $t = 1\,\text{s}$, so

$$0 = -2 + v_0 + \frac{1}{2} a \,.$$

The graph also shows that $x = 6\,\text{m}$ at $t = 2\,\text{s}$, so

$$6 = -2 + 2v_0 + 2a \,.$$

In these equations v_0 is in meters per second and a is in meters per second squared. Solve these two equations simultaneously for a. Use one of them to eliminate v_0 from the other.

CHAPTER 2 HINT FOR PROBLEM 36

(a) Place the origin of a coordinate system at the traffic light and take the positive x axis to be in the direction of travel of the vehicles. Take $t = 0$ to be the time the light changes to green. Then, the coordinate of the automobile as a function of time is given by

$$x_a(t) = \frac{1}{2}at^2$$

and the coordinate of the truck is given by

$$x_t(t) = v_t t\,.$$

The automobile overtakes the truck when $x_a = x_t$. Solve

$$\frac{1}{2}at^2 = v_t t$$

for t, the time when this occurs, then evaluate $x_t = v_t t$ to find where it occurs.

(b) Evaluate $v_a = at$.

(a) A collision is avoided if the passenger train slows to the speed of the locomotive before its coordinate is the same as that of the locomotive. Place the coordinate system with its origin at the front of the train when the engineer sees the locomotive and take the time to be zero at this instant. Let v_{0T} be the initial speed of the train and let a be its acceleration. Then, the coordinate of the train is given by

$$x_T = v_{0T}t + \frac{1}{2}at^2$$

and its velocity is given by

$$v_T = v_{0T} + at\,.$$

Let x_{0L} be the coordinate of the back of the locomotive when it is spotted by the engineer and let v_L be the velocity of the locomotive. The coordinate of the locomotive is then given by

$$x_L = x_{0L} + v_L t\,.$$

You want $v_T = v_L$ while $x_T < x_L$.

Solve

$$v_{0T} + at = v_L$$

for t. This gives the time when the velocities of the train and locomotive are the same, in terms of the acceleration of the train. Substitute this expression into

$$x_T = v_{0T}t + \frac{1}{2}at^2$$

and into

$$x_L = x_{0L} + v_L t\,.$$

Substitute the resulting expressions into $x_T < x_L$ and solve for the acceleration. You should get

$$a < -\frac{(v_L - v_{0T})^2}{2x_{0L}}$$

or, since a is negative

$$|a| > \frac{(v_L - v_{0T})^2}{2x_{0L}}\,.$$

The minimum magnitude of the acceleration that will do the job is given by

$$|a|_{\min} = \frac{(v_L - v_{0T})^2}{2x_{0L}}\,.$$

(b) The graph for the locomotive should be a straight line with a positive slope of 29.0 km/h, starting from $x = 676\,\mathrm{m}$ at $t = 0$. The graph for the train should be a parabolic curve. It starts at $x = 0$ with an initial slope of 161 km/h (much greater than that for the locomotive) but its slope should uniformly decrease, indicating a negative acceleration. The slope of the curve representing the train should just graze the line representing the locomotive and at that point it should have the same slope as the line.

(a) Take the y axis to be vertical with the upward direction positive and place the origin at the point where the sphere is dropped. Take the time t to be zero when the sphere is dropped. The initial velocity of the sphere is zero. The coordinate of the sphere is given by $y = -\frac{1}{2}at^2$. Set $y = -145\,\mathrm{m}$ and solve for t.

(b) The velocity of the sphere is given by

$$v = -gt\,.$$

Evaluate this for the value of t you found in part (a).

(c) Use

$$v^2 = v_0^2 + 2a\,\Delta y\,.$$

Set v equal to zero, v_0 equal to the velocity you found in part (b), and a equal to $+25g$. Solve for Δy. The distance traveled by the sphere is the magnitude of Δy.

Alternatively, you might solve

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

and

$$0 = v_0 + at$$

simultaneously for Δy. Use the second equation to eliminate t from the first.

Chapter 2 Hint for Problem 52

(a) When the fuel runs out, the acceleration changes from $4.00\,\mathrm{m/s^2}$ up to $9.8\,\mathrm{m/s^2}$ down. Work the problem in two parts: first find the altitude and velocity of the rocket when its fuel is gone, then use these values as initial conditions for the free-fall part of the motion.

Take the y axis to be positive in the upward direction and place the origin to be at the launch site. Set $t = 0$ at launch. Use

$$y = \frac{1}{2}at^2$$

and

$$v = at$$

to find the coordinate and velocity at the end of $6.00\,\mathrm{s}$. Here $a = 4.00\,\mathrm{m/s^2}$. You should get $72.0\,\mathrm{m}$ and $24.0\,\mathrm{m/s}$.

For the second part of the motion reset the time to 0 and set $y_0 = 72.0\,\mathrm{m}$ and $v_0 = 24.0\,\mathrm{m/s}$. Solve

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

and

$$v = v_0 - gt$$

for the coordinate when $v = 0$.

(b) Again take the initial conditions to be $y_0 = 72.0\,\mathrm{m}$ and $v_0 = 24.0\,\mathrm{m/s}$. Solve

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

for the time when $y = 0$. This is a quadratic equation and has two solutions, one positive and one negative. You want the positive solution. To the result, you must add the $6.00\,\mathrm{s}$ of the powered part of the motion.