CHAPTER 18 HINT FOR EXERCISE 4

Assume the soldiers at the rear of the column are one pace behind the drummer. Then the time for the drum sound to reach the rear of the column is (60 s)/(120) = 0.50 s. How far does sound travel in this time? Its speed is 343 m/s.

(a) The time taken by the wave that travels in air is $t_a = L/v$ and the time taken by the wave that travels along the pipe is $t_p = L/V$. Find an expression for the difference in the times: $t = t_a - t_p$.

(b) Solve the expression you found in part (a) to obtain an expression for L. Evaluate it using v = 343 m/s and V = 5941 m/s (see Table 18–1 of the text).

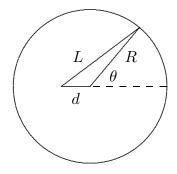
(a) The wavelength is $\lambda = 24$ cm and the frequency is f = 25 Hz. The wave speed is $v = \lambda f$.

(b) Write

$$y = y_m \sin(kx + \omega t + \phi)$$

for the displacement. Here y_m is the amplitude, k is the angular wave number, ω is the angular frequency, and ϕ is the phase constant. Notice that the term containing the coordinate x and the term containing the time t enter with the same sign because the wave is moving in the negative x direction. The amplitude is 0.30 cm, the angular wave number is $k = 2\pi/\lambda$, and the angular frequency is $\omega = 2\pi f$. Use the condition that y = 0 for x = 0 and t = 0 to find the value for the phase constant ϕ .

Draw a circle of radius R with one of the point sources at its center. Draw a line from this source to a point on the circle and let θ be the angle between the line and a coordinate axis, as shown on the diagram to the right. Place the other source on the coordinate axis, a distance $d (= 2.0\lambda)$ from the first source, and let L be the distance from the second source to the point on the circle. The difference in the distance traveled by the waves from the two sources is L - R.



Use the law of cosines to find an expression for L:

$$L = \sqrt{R^2 + 2Rd\cos\theta + d^2}$$

Use the binomial theorem to approximate this expression for R much greater than d. The result is $L \approx = R + d \cos \theta$.

(a) For constructive interference L - R must be an integer number of wavelengths. Thus

$$d\cos\theta = n\lambda\,,$$

where n is a positive or negative integer or zero. Replace d with 2λ and find the number of different values of θ that satisfy this condition.

(b) For destructive interference L - R must be an odd number of half wavelengths. Thus

$$d\cos\theta = n\lambda/2$$
,

where n is a positive or negative odd integer. Replace d with 2λ and find the number of different values of θ that satisfy this condition. As an alternative, you might convince yourself that there must be a point of completely destructive interference between every two adjacent points of constructive interference.

(a) The phase difference of the two waves at the listener is $\phi = k \Delta x$, where k is the angular wave number of the waves and Δx is the difference in the distance they travel from the speakers to the listener. For a minimum signal ϕ must be an odd multiple of π or Δx must be an odd multiple of $\lambda/2$. Thus $\Delta x = n\lambda/2$ or

$$\lambda = \frac{2\,\Delta x}{n}\,,$$

where n is an odd integer. The frequencies are given by

$$f = \frac{v}{\lambda} = \frac{nv}{2\,\Delta x}\,.$$

The three lowest occur for n = 1, 3, and 5. Use v = 343 m/s and $\Delta x = 19.5$ m - 18.3 m = 1.2 m.

(b) For a maximum signal the phase difference must be a multiple of 2π . Thus $\Delta x = n\lambda$, where n is an integer, and

$$f = \frac{nv}{\Delta x}$$

The three lowest frequencies occur for n = 1, 2, and 3.

ans: (a) 143 Hz, 429 Hz, 715 Hz; (b) 286 Hz, 572 Hz, 858 Hz

Consider a cylinder with radius r and length L concentric with the train and with its central axis along the train. If P_s is the rate with which the train emits energy in the form of sound, then the sound intensity at the surface of the cylinder is $I = P_s/A$, where A is the area of the curved portion of the cylinder surface. Use $A = 2\pi rL$ and recall that the intensity is proportional to the square of the wave amplitude.

(a) Use the equation

$$\beta = (10 \,\mathrm{dB}) \log \frac{I}{I_0}$$

for the sound level associated with a wave of intensity I. Here I_0 is the reference level. The difference in sound levels of two waves is

$$\beta_2 - \beta_1 = (10 \,\mathrm{dB}) \left[\log(I_2/I_0) - \log(I_1/I_0) \right] = (10 \,\mathrm{dB}) \log(I_2/I_1).$$

Solve for the ratio I_2/I_1 . You should get $I_2/I_1 = 10^u$, where $u = (\beta_2 - \beta_1)/(10 \text{ dB})$.

(b) and (c). Recall that the intensity is proportional to the square of the pressure amplitude and is also proportional to the particle displacement amplitude.

The sphere centered at the point source and with the microphone on its surface has a surface area that is given by $A = 4\pi r^2$, where r is the distance from the source to the microphone. The intensity at the microphone is

$$I = \frac{P_{\rm avg}}{A} = \frac{P_{\rm avg}}{4\pi r^2} \,,$$

where P_{avg} is the average power output of the loudspeaker. The average power intercepted by the microphone is given by $P_{\text{avg }m} = IA_m$, where A_m is the cross-sectional area of the microphone.

(a) Assume the string vibrates in its fundamental standing wave mode for each note. The wavelength for an A note is $\lambda_A = 2L$, where L is the length of the string. The frequency is

$$f_A = \frac{v}{\lambda} = \frac{v}{2L} \,,$$

where v is the wave speed on the string. The wavelength for a C note is $\lambda_C = 2x$ and the frequency is v/2x, where x is the length of the vibrating portion of the string when the finger is placed on it. The ratio is $f_C/f_A = L/x$, Solve for x.

(b) The ratio of wavelengths is $\lambda_A/\lambda_C = L/x$.

(c) The wavelength in air for the A note is $\lambda_A = v/f_A$ and the wavelength in air for the C note is $\lambda_C = v/f_C$, where v is now the speed of sound in air. The frequencies are the same as the frequencies of the vibrating string. The ratio is $\lambda_A/\lambda_C = f_C/f_A$.

(a) Since one end of the tube is open and the other end is closed the tube length L_T is related to the wavelength λ by

$$L_T = \frac{n}{\lambda/4} \,,$$

where n is an odd integer. Since the air column in the tube is vibrating in its fundamental mode, n = 1 and $\lambda = 4L_T$. The frequency is given by

$$f = \frac{v}{\lambda} = \frac{v}{4L_T} \,,$$

where v is the speed of sound in air (343 m/s).

(b) The speed of waves on the wire is given by

$$v = \sqrt{\frac{ au}{\mu}},$$

where τ is the tension in the wire and μ is the linear density of the wire. It is also given by $v = \lambda f$, where λ is the wavelength and f is the frequency of a traveling sinusoidal wave on the wire. Thus

$$\lambda f = \sqrt{\frac{\tau}{\mu}}$$

or

$$\tau = \lambda^2 f^2 \mu \,.$$

The given mass and length of the wire can be used to calculate μ . Use the standing wave condition to calculate λ . Since both ends of the wire are fixed the wavelength λ on the wire is related to the wire length L_W by $L_W = n\lambda/2$, where n is an integer. Since the wire is vibrating in its fundamental mode, n = 1 and $\lambda = 2L_W$.

(a) For a pipe that is open at both ends an integer number of half wavelengths must fit into its length, so the wavelength for pipe A is $\lambda_A = 2L_A/n$, where L_A is the length of pipe A and n is an integer. Since air in the pipe is vibrating in its third lowest harmonic, set n equal to 3. Draw a diagram of the displacement amplitude as a function of position along the pipe and convince yourself that nodes occur at $\lambda_A/4$, $3\lambda_A/4$, and $5\lambda_A/4$.

(b) For a pipe that is open at one end and closed at the other an odd number of quarter wavelengths must fit into its length, so the wavelength for pipe B is $\lambda_B = 4L_B/n$, where L_B is the length of pipe B and n is an odd integer. Since the air in pipe B is vibrating in its second lowest harmonic set n = 3. The vibration frequency for pipe A is $f_A = v/\lambda_A = 3v/2L_A$ and the vibration frequency for pipe B is $f_B = 3v/4L_B$. Equate the expressions for the frequencies of the two pipes to each other and solve for L_B .

(c) Now the air in pipe A is vibrating in its fundamental, so the wavelength is $\lambda_A = 2L_A$. The frequency is

$$f_A = \frac{v}{\lambda} \,.$$

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Since there are four beats per second, the violin string is oscillating at either 444 Hz or at 436 Hz. Since the string is a little too taut the wave speed is a little too high. This means that either the wavelength or the frequency or both are a little too high. Since the wavelength is determined by the distance between the fixed ends and is not affected by the tension in the string, it must be that the frequency is too high. It must be 444 Hz. To find the period, calculate the reciprocal of the frequency.

Think of two separate parts to the problem. In the first part a stationary source emits a wave of frequency f that is received by the moving truck. Calculate the frequency f' received by the truck. Use the Doppler shift equation

$$f' = f \, \frac{v \pm v_D}{v \pm v_S} \,,$$

where v is the speed of sound, v_D is the speed of the detector (the truck) and v_S is the speed of the source. Set v = 343 m/s, $v_D = 45 \text{ m/s}$, and $v_S = 0$. Since the truck is approaching the source, f' is greater than f and you must use the plus sign in the numerator.

In the second part, the truck sends out a wave with frequency f' (the reflected wave). It is received back at the stationary detector. Its frequency is f''. Now

$$f'' = f' \, \frac{v \pm v_D}{v \pm v_S} \,,$$

Set $v_D = 0$ and $v_S = 45$ m/s. The frequency f'' is greater than f' so you must use the minus sign in the denominator.

The Mach cone angle is given by

$$\sin\theta = \frac{v}{v_S}\,,$$

where v is the speed of sound and v_S is the speed of the plane. The altitude of the plane is given by

$$h = v_S t \tan \theta \,.$$

Find the value of θ , then calculate h.

(a) Since the source is moving toward the listener and the listener is moving toward the source in still air, the frequency heard is given by

$$f' = f \frac{v + v_D}{v - v_S} \,,$$

where v_S is the speed of the train with the whistle, v_D is the speed of the other train, and v is the speed of sound in air (343 m/s).

(b) Use the same equation but take the train speeds to be those relative to the air. That is, the speed of the train with the whistle is $v_S = 30.5 \text{ m/s} + 30.5 \text{ m/s} = 61.0 \text{ m/s}$ and the speed of the other train is $v_D = 0$.

(c) Now, $v_S = 0$ and $v_D = 61.0 \text{ m/s}$.

[ans: (a) 598 Hz; (b) 608 Hz; (c) 589 Hz]