Chapter 14 Hint for Exercise 6

Both Earth and its moon pull on the spaceship, but in opposite directions. Write an equation for the net force in terms of the distance from Earth, equate the net force to zero, and solve for the distance. Look in an appendix for the values of the mass of Earth, the mass of the moon, and the Earth-moon distance.

Let M_m be the mass of the moon, M_e be the mass of Earth, and M_s be the mass of spaceship. If R is the center-to-center distance from Earth to the moon and r is the distance from the center of Earth to the spaceship, then the magnitude of the net force on the spaceship is given by

$$F = GM_s \left[\frac{M_e}{r^2} - \frac{M_m}{(R-r)^2} \right] .$$

$F = 0$ yields a quadratic equation for r:

$$(M_e - M_m)r^2 - 2M_e Rr + M_e R^2 = 0 \, .$$

Solve for r. You will get two solutions. Reject the one that corresponds to the far side of the moon.

CHAPTER 14 HINT FOR PROBLEM 9

Vectorially sum the four forces acting on the central mass, one due to each of the masses at the corners. Notice that two of the corners are occupied by identical masses, the same distance from the center and along the same line but on opposite sides. The forces due to these two masses cancel. The other masses pull on the central mass in opposite directions. Use

$$F = \frac{Gmm_5}{r^2},$$

where $r = a/\sqrt{2}$ and a is the edge length, to compute each individual force.

[ans: $1.7 \times 10^{-2}\,\mathrm{N}$, toward the $300\,\mathrm{kg}$ mass]

Chapter 14 Hint for Exercise 14

Your weight at the bottom is given by

$$W_b = \frac{GMm}{R^2}$$

and your weight at the top is given by

$$W_t = \frac{GMm}{(R+h)^2}\,,$$

where M is the mass of Earth, m is your mass, R is the radius of Earth, and h is the altitude of the tower. Notice that $W_t = W_b R^2/(R+h)^2$ and the change in your weight is given by

$$\Delta W = W_t - W_b = W_b\left[1 - \frac{R^2}{(R+h)^2}\right]\,.$$

Because R and $R+h$ differ only in the fifth significant figure there may be round-off error if you evaluate this expression as it is written. Put both terms in the brackets over the same denominator and cancel as much of the numerator as you can. You should obtain

$$W_t - W_b = W_b\frac{2Rh + h^2}{(R+h)^2}\,.$$

Evaluate this expression. Look up the radius of Earth in Appendix C.

Chapter 14 Hint for Problem 18

Use the shell theorems. The magnitude of the force on a particle a distance r from the center of the planet is the same as the force would be if all the mass inside a sphere of radius r were located at the center of the planet. In (a) this is M and in (b) it is $5M$. According to Newton's second law, the magnitude of the acceleration is the magnitude of the force divided by the mass of the particle.

Chapter 14 Hint for Problem 21

Assume material on the surface just barely goes around with the star. Then, the normal force of the surface on the material is zero and the force of gravity is just sufficient for uniform circular motion. Let m be the mass of the material, M be the mass of the star, and R be the radius of the star. Equate the gravitational force GmM/R^2 to the product of the mass and acceleration of the material $mR\omega^2$ and solve for M. You must convert $1\,\mathrm{rev/s}$ to rad/s.

$\Big[$ans: $4.7 \times 10^{24}\,\mathrm{kg}\Big]$

Chapter 14 Hint for Exercise 22

Sum the forces due to each shell individually. If the particle is outside a shell the force of that shell is as if the entire mass of the shell were at its center. If the particle is inside a shell the force of that shell is zero. In (a) the particle is outside both shells; in (b) it is outside the shell of mass M_1 and inside the shell of mass M_2; and in (c) it is inside both shells.

Chapter 14 Hint for Problem 23

(a) and (b) The first point is outside the sphere, so the force is given by

$$F = \frac{GMm}{r^2},$$

where M is the mass of the sphere and r is the distance from the center of the sphere to the particle. The second point is inside the sphere, so you need consider only the mass that is closer to the sphere center than the particle. This is $M(r^3/R^3)$, where R is the radius of the sphere. Thus

$$F = \frac{GMmr}{R^3}.$$

(c) Obtain an expression for the mass within the sphere and closer to the center than r and substitute this expression into Newton's law of gravity.

[ans: (a) $(3.0 \times 10^{-7}\,\mathrm{N/kg})m$; (b) $(3.3 \times 10^{-7}\,\mathrm{N/kg})m$; (c) $(6.7 \times 10^{-7}\,\mathrm{N/kg \cdot m})mr$]

Chapter 14 Hint for Exercise 26

In Exercise 1 a 5.2-kg particle and a 2.4-kg particle are separated by 19 m. Their potential energy (relative to a potential energy of zero at infinite separation) is given by

$$U = -G\frac{m_1 m_2}{r}\,,$$

where r is their separation. If the separation is tripled the change in the potential energy is

$$\Delta U = -\frac{G m_1 m_2}{r}\left[\frac{1}{3} - 1\right] = \frac{2 G m_1 m_2}{3r}\,.$$

The work done by the gravitational force is $-\Delta U$ and the work done by you is $+\Delta U$.

(a) Let K_1 be the kinetic energy of the rocket when the engine shuts off and let U_1 be the potential energy of the Earth-rocket system then. Let K_2 be the kinetic energy of the rocket when it is 1000 km above Earth and let U_2 be the potential energy of the Earth-rocket system then. Mechanical energy is conserved, so

$$K_1 + U_1 = K_2 + U_2\,.$$

Now $K_1 = \frac{1}{2}mv_1^2$, where m is the mass of the rocket and v_1 is its speed when the engine shuts off. The initial potential energy is $U_1 = -GmM/r_1$, where M is the mass of Earth and r_1 is the distance from the center of Earth to the rocket when the engine shuts off. This is the radius of Earth plus the altitude of the rocket. The final potential energy is $U_2 = -GmM/r_2$, where r_2 is the distance from the center of Earth to the rocket when it is 1000 km above Earth.

Solve

$$\frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}$$

for K_2. You can find values for the mass and radius of Earth in Appendix C of the text.

(b) When the rocket is at its maximum altitude its kinetic energy is zero. Let r_3 be the distance from the center of Earth to the rocket then. Solve

$$\frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = -\frac{GmM}{r_3}$$

for r_3. Subtract the radius of Earth.

Chapter 14 Hint for Problem 42

Use the law of periods for planets and satellites:

$$T^2 = \frac{4\pi^2}{GM} r^3 \,,$$

where T is the period, M is the mass of the central body (Earth) and r is the radius of the orbit. Solve for M

Chapter 14 Hint for Exercise 46

According to Fig. 14–13 the distance between foci is $2ea$, where e is the eccentricity and a is the semimajor axis. Compute this distance and divide by the solar radius.

Chapter 14 Hint for Problem 52

The magnitude of the acceleration due to gravity at the surface of planet with mass M and radius R is

$$a_g = \frac{GM}{R}\,.$$

This equation is to be solved for R, once M is known.

Apply the law of periods to the satellite:

$$T^2 = \frac{4\pi^2}{GM} r^3\,,$$

where T is the period, r is the radius of the orbit, and M is the mass of the planet. Solve this equation for M and use the value in the equation for the acceleration due to gravity.

Chapter 14 Hint for Problem 54

Each of the stars of mass m is pulled toward the center of the orbit by the gravitational force of the other star of mass m and by the central star. Since the other star of mass m is $2r$ distant and the star of mass M is r distant the net force has magnitude

$$F = \frac{GMm}{r^2} + \frac{Gm^2}{4r^2}\,.$$

This must equal mv^2/r. Furthermore, the period T and speed v are related by $v = 2\pi r/T$. Thus

$$\frac{GMm}{r^2} + \frac{Gm^2}{4r^2} = \frac{m(2\pi r/T)^2}{r}\,.$$

Solve for T.

CHAPTER 14 HINT FOR PROBLEM 58

(a) The potential energy of the system consisting of a massive central body and a satellite in a circular orbit around it is given by

$$U = -\frac{GmM}{r}\,,$$

where m is the mass of satellite, M is the mass of the central body, and r is the radius of the orbit. The requested ratio is

$$\frac{U_B}{U_A} = \frac{r_A}{R_B}\,.$$

The kinetic energy of the satellite is given by

$$K = \frac{GmM}{2r}$$

and the requested ratio is

$$\frac{K_B}{K_A} = \frac{r_A}{r_B}\,.$$

(c) The mechanical energy of the Earth-satellite system is given by

$$E = -\frac{GmM}{2r}\,.$$

The difference in mechanical energies is

$$E_B - E_A = \frac{GmM}{2}\left[\frac{1}{r_A} - \frac{1}{R_B}\right]\,.$$