Chapter 13 Hint for Exercise 6

The sum of the torques acting on the scaffold must be zero. To answer part (a) place the origin at the far end of the scaffold and to answer part (b) place it at the near end. Then each torque equation contains only one unknown, the force exerted by the cable at the end opposite the origin. Once you have solved part (a) you can also use the condition that the sum of the forces on the scaffold is zero to obtain the force of the cable at the far end.

Each cable exerts a (different) upward force on the scaffold. The force of gravity on the scaffold for our purposes acts at the center of the scaffold. The man exerts downward force at the point where he stands. Its magnitude equals the weight of the man.

(a) Let L be the length of the scaffold and M be its mass. Let m be the mass of the man and ℓ be the distance from the man to the near end of the scaffold. If T_n is the force of the near cable and the origin is at the far end of scaffold, then

$$T_n L - \frac{MgL}{2} - mg\ell = 0\,.$$

Solve for T_n.

(b) If T_f is the force of the far cable and the origin is at the near end of the cable, then

$$T_f L - \frac{MgL}{2} - mg(L-\ell) = 0\,.$$

Solve for T_f. Alternatively, the sum of forces equation is

$$T_n + T_f - Mg - mg = 0\,.$$

This can also be solved for T_f once T_n is known.

Chapter 13 Hint for Exercise 10

Consider the forces acting on the half of the rope from the man to the car. The man exerts a force on it, the car exerts a force on it, and the other half of the rope exerts a force on it. The last two forces are along the rope at their points of application. Take the x axis to be along the original direction of the rope and the y axis to be along the direction of the force exerted by the man. Write the equations for the vanishing of the sum of the x components of the force and for the vanishing of the y components. Solve for the magnitude of the force exerted on the rope by the car. This is the same as the magnitude of the force exerted by the rope on the car. You will need to find the angle between the rope and the straight-ahead direction. Use the right triangle with the rope as the hypotenuse.

Chapter 13   Hint for Exercise 12

When the crate is about to tip, both the normal force of the floor and the horizontal force of the obstruction are applied at the edge in contact with the obstruction. In addition, a force of gravity acts at the center of the crate. Calculate the position at which the 350 N force must be applied to hold the crate in equilibrium.

Draw a force diagram for the crate. Show it in cross section, as a square, with the obstruction at the lower right corner and the applied force F acting horizontally a distance d from the floor at the left face. Show the normal force N of the floor acting upward at the lower right corner, the force F_o of the obstruction acting to the left at the lower right corner, and the force of gravity W acting downward at the center.

If you pick the lower right corner as the origin, all you need to solve the problem is the condition for rotational equilibrium:

$$WL/2 - Fd = 0\,,$$

where L is the length of one side of the cube. Solve for d.

Chapter 13 Hint for Problem 16

Consider the equilibrium equations for each of the two knots.

Draw a force diagram for the knot on the left. Show the forces of the three strings that are tied there. Two of the tension forces are unknown. The third equals the weight W_1 of the mass hung from the knot. If the x axis is horizontal and the y axis is vertical, the equilibrium equations are

$$T_1 \cos 35^\circ - W_1 = 0$$

and

$$T_1 \sin 35^\circ - T_3 = 0\,.$$

All torques about the knot vanish.

Draw a force diagram for the knot on the right. Show the forces of the three strings tied there. One tension force equals the weight W_2 of the mass hung there; the others are unknown. The angle θ is also unknown. The equilibrium equations are

$$T_2 \sin\theta - T_3 = 0$$

and

$$T_2 \cos\theta - W_2 = 0\,.$$

Again all torques about the knot vanish.

The equations for the left knot can be solved for T_1 and T_3, then the equations for the right knot can be solved for T_2 and θ. Use $\cos^2\theta + \sin^2\theta = 1$ to eliminate θ when you solve for T_2. Divide one equation by the other to eliminate T_2 and obtain an equation for $\tan\theta$.

Chapter 13 Hint for Problem 19

Use the conditions for equilibrium of the structure.

Draw a force diagram for the structure. Show all the forces in the diagram of the text. Since the net force on the structure vanishes,

$$F_v - F_1 - F_2 = 0$$

and

$$F_h - F_3 = 0\,.$$

Since the net torque on the structure vanishes,

$$F_v(d+c) - F_1c - F_2(b+c) - F_3a = 0\,,$$

where the origin was taken to be at the left end of the structure. Solve the first equation for F_v, the second for F_h, and the third for d.

[ans: (a) 5.0 N; (b) 30 N; (c) 1.3 m]

Chapter 13 Hint for Problem 20

(a) Solve the equilibrium condition equations for the rod.

Draw a force diagram for the rod. The wall exerts a vertical force F_v and a horizontal force F_h. Take the vertical force to be upward and the horizontal force to be to the right. The sign exerts a force equal to its weight $m_s g$, which may be considered to be applied at the mid-point of its length, a distance $L - \ell/2$ from the left end of the rod. Here L is the length of the rod and ℓ is the length of the sign. Include the tension force of the cable, acting at the end of the rod and at an angle θ above the horizontal. The rod is essentially massless, so the force of gravity on it is ignored. A little trigonometry shows that if d is the distance between the left end of the rod and the point where the cable is attached, then $\sin\theta = d/\sqrt{d^2 + L^2}$ and $\cos\theta = L/\sqrt{d^2 + L^2}$.

Take the x axis to be horizontal and the y axis to be vertical. The conditions for equilibrium are then

$$T\sin\theta + F_v - m_s g = 0\,,$$

$$F_h - T\cos\theta = 0\,,$$

and

$$TL\sin\theta - m_s g(L - \ell/2) = 0\,.$$

The last equation can easily be solved for T.

(b) and (c) The first of the equilibrium equations can be solved for F_v and the second can be solved for F_h.

(a) Use the equilibrium conditions for the rock climber. Since he is on the verge of slipping the frictional force of the rock on his hands has magnitude given by $f_1 = \mu_1 N_1$ and the frictional force of the rock on his feet has magnitude given by $f_2 = \mu_2 N_2$, where N_1 is the (horizontal) normal force of the rock on his hands and N_2 is the (horizontal) normal force of the rock on his feet.

Draw a force diagram for the rock climber. Show the two normal forces, the one on his hands to the right and the one on his boots to the left, and the two frictional forces, both upward. Also show the force of gravity mg downward. For practical purposes, this acts at the center of mass of the climber. Take the x axis to be horizontal and the y axis to be vertical. Since the net force on the climber vanishes

$$\mu_1 N_1 + \mu_2 N_2 - mg = 0$$

and

$$N_1 - N_2 = 0\,.$$

Take the origin to be at his boots. Then the condition for rotational equilibrium becomes

$$N_1 h + \mu_1 N_1 w - mg(d + w) = 0\,,$$

where h is the vertical distance from his boots to his hands.

The first two equations can be solved for N_1 and N_2. The second tells you that $N_1 = N_2$. The first then yields

$$N_1 = \frac{mg}{\mu_1 + \mu_2}\,.$$

(b) Solve the third equilibrium condition equation for h. You should get

$$h = \frac{mg(d + w) - \mu_1 N_1 w}{N_1}\,.$$

(c) You derived an algebraic expression for N_1. What happens to its value if the friction coefficients are reduced? To find out what happens to h, substitute the expression for N_1 into the expression you obtained for h. You should get

$$h = \mu_1(d + w) + \mu_2(d + w)\,.$$

What happens when the coefficients are reduced?

Chapter 13 Hint for Problem 24

For each brick the sum of the forces on it is zero and the sum of the torques on it is also zero. Assume each brick is on the verge of toppling. Then the upward normal force of the brick below acts the right edge of the brick below. The force of gravity, for practical purposes, acts the center of the brick. Don't forget to include the downward normal force of the brick above, if any. In each case place the origin for computing torques at the right end of the brick. start with the highest brick.

Chapter 13 Hint for Problem 28

The sum of the forces acting on the bar is zero and the sum of the torques is also zero. The forces on the bar are the force of the hinge, at the left end of the bar, the downward force W of the movable weight, at a distance x from the left end, and the tension force of the wire, at the right end. Take the vertical component of the hinge force to be upward and the horizontal component to be rightward. Then the equations for translational equilibrium are

$$F_v - W + T\sin\theta = 0$$

and

$$F_h - T\cos\theta = 0\,.$$

Place the origin at the hinge. Then the equation for rotational equilibrium is

$$TL\sin\theta - Wx = 0\,.$$

The third equation can be solve for T, the first can be solved for F_v and the second can be solved for F_h.

CHAPTER 13 HINT FOR PROBLEM 30

(a) Consider the conditions for equilibrium of the bar with W placed an arbitrary distance x from the left end. Take the tension force of the wire to be the maximum, 500 N, and calculate x.

Draw a force diagram for the bar. Show W acting downward a distance x from the left end; the force of gravity W_b acting downward at the center of the bar, a distance $L/2$ from the left end; the wall pushing toward the right with a force F_h and upward with a force F_v; and the tension force of the wire, at an angle θ above the horizontal.

Take the x axis to be horizontal and the y axis to be vertical. The horizontal components of the forces sum to zero:

$$F_h - T\cos\theta = 0\,.$$

The vertical components sum to zero:

$$F_v + T\sin\theta - W - W_b = 0\,.$$

The torques sum to zero:

$$TL\sin\theta - Wx - W_bL/2 = 0\,,$$

where the left end of the rod was taken to be the origin. Solve the third equation for x.

(b) and (c) Solve the two force equations for F_h and F_v.

CHAPTER 13 HINT FOR PROBLEM 35

For each value of the coefficient μ_s find the angle at which sliding occurs and the angle at which tipping occurs. The smaller of the two determines which event occurs.

Consider sliding first. Draw a force diagram for the crate. The force of gravity mg acts downward, the normal force N acts perpendicular to the plane, and the force of friction f acts parallel to the plane, up the plane. Take the x axis to be down the plane and the y axis to be perpendicular to the plane. Newton's second law yields

$$mg\sin\theta - f = 0$$

and

$$mg\cos\theta - N = 0\,.$$

Since the crate is on the verge of sliding $f = \mu_s N$. Eliminate N and solve for θ. You should get

$$\tan\theta = \mu_s\,.$$

For each of the two values of μ_s find the value of θ at which sliding starts. You should get 31° for $\mu_s = 0.60$ and 35° for $\mu_s = 0.70$.

Now suppose the crate is on the verge of tipping over. Draw a force diagram. The normal force N and the friction force f now act at the front edge of the crate, the force of gravity acts at the center of gravity, a distance d ($= 0.90\,\mathrm{m}$) from the bottom face and a distance $L/2$ from the front face. Here L is the length of a cube edge. Place the origin at the front edge and write the equation for rotational equilibrium:

$$mgd\sin\theta - mg(L/2)\cos\theta = 0\,.$$

Solve for θ. You should get 34°.

Now decide which event occurs for each value of μ_s.

[ans: (a) sliding occurs, at $\theta = 31^\circ$; (b) tipping occurs, at $\theta = 34^\circ$]

(a) and (b) The force exerted by cylinder A is given by

$$F_A = \frac{E_A A_A \, \Delta L_A}{L_A} \, ,$$

where E_A is the Young's modulus, A_A is the cylinder's cross-sectional area, L_A is the length of the cylinder, and ΔL_A is the amount by which the cylinder is compressed when the brick is in place. Similarly the force exerted by cylinder B is given by

$$F_B = \frac{E_B A_B \, \Delta L_B}{L_B} \, .$$

In addition, the brick is in equilibrium, so $F_A + F_B = W$, where W is the weight of the brick. The cylinders start with the same length and the brick is horizontal, so $L_A = L_B$ and $\Delta L_A = \Delta L_B$. Use the conditions $A_A = 2A_B$ and $E_A = 2E_B$, then solve for F_A and F_B.

First calculate the ratio

$$\frac{F_A}{F_B} = \frac{E_A A_A}{E_B A_B} \, .$$

You should get 4. Next substitute $F_A = 4F_B$ into $F_A + F_B = W$ and solve for F_B. Multiply by 4 to obtain F_A.

(c) Take the forces of the cylinders to act at the center lines. The brick is in rotational equilibrium, so

$$F_A d_A = F_B d_B \, ,$$

where the origin was taken to be at the center of mass of the brick. Solve for d_A/d_B.

(a) The mass of the material above the tunnel is given by $M = \rho V$, where ρ is the density of the material and V is its volume. The weight of this material is $W = Mg$.

(b) If there are N columns and each exerts a force F, then $NF = W$. If U is the ultimate strength of steel ($400 \times 10^6\,\mathrm{N/m^2}$) and A is the cross-sectional area of each column, then $F = UA/2$. Solve $NUA/2 = W$ for N. Round up to the nearest integer.