(a) and (b) Find an expression for the angular velocity ω as a function of time by differentiating $\theta(t)$. Evaluate the expression for the two values of the time. The angular velocity in radians per second is given by $\omega(t) = 4.0 - 6.0t + 3.0t^2$.

(c) Use the definition of the average angular acceleration:

$$\alpha_{\text{avg}} = \frac{\omega_f - \omega_i}{\Delta t} \,,$$

where ω_i is the angular velocity at the beginning of the interval and ω_f is the angular velocity at the end..

(d) and (e) Differentiate $\omega(t)$ to obtain an expression for the angular acceleration as a function of time. Evaluate the expression for the two values of the time. You should get $\alpha(t) = -6.0 + 6.0t$ for the angular acceleration in radians per second squared.

Chapter 11 Hint for Problem 6

(a) The minimum speed will just allow the arrow to travel its own length in the time the wheel turns through one-eighth of a revolution. This time is given by

$$t = \frac{\theta}{\omega}\,,$$

where $\theta = 1/8\,\text{rev}$. The minimum speed of the arrow is given by

$$v = \frac{\ell}{t} = \frac{\omega\ell}{\theta}\,,$$

where ℓ is the length of the arrow.

(b) Near the rim there is more space but the spokes are moving faster than near the axle. In fact, both the spoke speed and the arc length between spokes are proportional to the distance from the axle.

Chapter 11 Hint for Exercise 8

The angular acceleration is constant so the equations for rotation with constant angular acceleration can be used.

(a) Use $\omega = \omega_0 + \alpha t$. Keep the angular velocities in revolutions per minute and convert $12\,\mathrm{s}$ to minutes.

(b) Use $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

The wheel rotates with constant angular acceleration.

(a) The line will turn with the wheel in the positive direction until its angular velocity is zero. Then θ is a maximum. Solve

$$0 = \omega_0 + \alpha t$$

for t, then evaluate

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2\,.$$

As an alternative, put $\omega = 0$ in

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

and solve for θ.

(b) and (c) Solve

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

for t. This is a quadratic equation and you should get two solutions for each value of θ.

The general solution to the quadratic equation is

$$t = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\alpha\theta}}{\alpha}\,.$$

Evaluate this expression for $\theta = 22\,\mathrm{rad}$ and for $\theta = -10.5\,\mathrm{rad}$.

[ans: (a) 44 rad; (b) 5.5 s, 32 s; (c) −2.1 s, 40 s]

Chapter 11 Hint for Problem 16

The disk rotates with constant angular acceleration.

(a) Solve

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

for α. Substitute $\omega = 15\,\mathrm{rev/s}$, $\omega_0 = 10\,\mathrm{rev/s}$, and $\theta = 60\,\mathrm{rev}$.

(b) Solve

$$\omega = \omega_0 + \alpha t$$

for t. Use the value for α found in the part (a).

(c) Solve

$$\omega = \omega_0 + \alpha t$$

for t. Now $\omega = 10\,\mathrm{rev/s}$ and $\omega_0 = 0$. The angular acceleration has the same value as before.

(d) Use

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 ,$$

with $\omega_0 = 0$. The time has the value found in part (c).

(a) Differentiate $\theta = 0.30t^2$ to obtain an expression for the angular velocity in radians per second as a function of time. Evaluate the expression for $t = 5.0\,\mathrm{s}$.

(b) Use $v = \omega r$, where r is the radius of the centrifuge and ω is the answer to part (a).

(c) Differentiate the angular velocity with respect to time to find the value of the angular acceleration, then use $a_t = \alpha r$ to find the tangential component of the acceleration.

(d) Use $a_r = \omega^2 r$ to find the radial component of the acceleration.

[ans: (a) $3.0\,\mathrm{rad/s}$; (b) $30\,\mathrm{m/s}$; (c) $6.0\,\mathrm{m/s^2}$; (d) $90\,\mathrm{m/s^2}$]

(a) Since the angular acceleration α is constant the angular velocity ω is given by

$$\omega = \omega_0 + \alpha t\,.$$

Solve for α and substitute $\omega_0 = 150\,\text{rev/min}$, $\omega = 0$, and $t = 2.2\,\text{h}$. You will get the angular acceleration in revolutions per hour-squared and you must convert to revolutions per minute-squared. The negative result for α indicates that the direction of the angular acceleration is opposite that of the initial angular velocity.

(b) Use

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2\,.$$

Be careful to use consistent units.

(c) Use $a_t = \alpha r$, where r is the distance from the rotation axis to the particle. Remember that α must be in radians per second-squared. You must convert the value you found in part (a). Use $1\,\text{rev} = 2\pi\,\text{rad}$ and $1\,\text{min} = 60\,\text{s}$.

(d) The radial acceleration is given by $a_r = \omega r^2$, where ω must be in radians per second. You must convert $75\,\text{rev/min}$. Since the radial and tangential accelerations are perpendicular to each other, the magnitude of the net linear acceleration is $a = \sqrt{a_r^2 + a_t^2}$.

Chapter 11 Hint for Problem 34

The translational kinetic energy is given by

$$K_{\text{com}} = \frac{1}{2} m v_{\text{com}}^2 ,$$

where m is the mass of the molecule and v_{com} is the speed of its center of mass. The rotational kinetic energy is $K_{\text{rot}} = (2/3) K_{\text{com}}$. It is given by

$$K_{\text{rot}} = \frac{1}{2} I \omega^2 ,$$

where I is the rotational inertia of the molecule and ω is its angular speed. Solve

$$\frac{1}{2} I \omega^2 = (2/3) \frac{1}{2} m v_{\text{com}}^2$$

for ω.

Chapter 11 Hint for Exercise 38

(a) According to Table 11–2, the rotational inertia of a thin rod that is rotating about one end is $I = (1/3)ML^2$, where L is the length of the rod and M is its mass. The helicopter rotor assembly contains three such rods, so the total rotational inertia is three times the rotational inertia of a single rod.

(b) The rotational kinetic energy is given by

$$K = \frac{1}{2} I_{\text{total}} \omega^2 \,,$$

where ω is the angular speed in radians per second. You must convert $350\,\text{rev/min}$. Use $1\,\text{rev} = 2\pi\,\text{rad}$ and $1\,\text{min} = 60\,\text{s}$.

(a), (b), and (c) The rotational inertia of a collection of particles is given by the sum

$$\sum m_i r_i^2 \,,$$

where m_i is the mass of particle i and r_i is its distance from the axis of rotation. If the axis of rotation is the x axis, then $r_i^2 = y_i^2 + z_i^2 = y_i^2$. The last equality holds for the four particles of this system because all are in the xy plane. If the axis of rotation is the y axis, then $r_i^2 = x_i^2 + z_i^2 = x_i^2$ and if the axis of rotation is the z axis, then $r_i^2 = x_i^2 + y_i^2$.

(d) Notice that

$$A = \sum m_i y_i^2 \,,$$

$$B = \sum m_i x_i^2 \,,$$

and the answer to part (c) is

$$\sum m_i (x_i^2 + y_i^2) \,.$$

(a) Use

$$K = \frac{1}{2} I \omega^2$$

to calculate the kinetic energy. Here I is the rotational inertia of the flywheel and ω is its angular speed. According to Table 11–2 $I = \frac{1}{2} M R^2$.

(b) Use

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} \,,$$

where ΔK is the change in the kinetic energy of the flywheel in time Δt. Since the flywheel starts fully charged and ends at rest, ΔK is the kinetic energy found in part (a). Solve for Δt.

Chapter 11 Hint for Exercise 46

Use

$$\tau = F\ell \sin\phi \, ,$$

where ℓ is the length of the pedal and ϕ is the angle between the pedal and the force. Since the force is downward this is the same as the angle made by the pedal with the vertical.

CHAPTER 11 HINT FOR PROBLEM 48

Use

$$\tau = F_t \ell \,,$$

where F_t is the tangential component of the force and ℓ is the distance from O to the point of application of the force. Attach appropriate signs to the torques and sum them.

For force A the tangential component is $F_A \cos(135° - 90°)$, for force B the tangential component is F_B, and for force C the tangential component is $F_C \cos(160° - 90°)$. Forces A and C tend to turn the body counterclockwise, so τ_A and τ_C are positive; force B tends to turn the body clockwise, so τ_B is negative. Sum the torques, with their signs, to obtain the net torque.

Chapter 11 Hint for Exercise 52

Use

$$\tau_{\text{net}} = I\alpha\,,$$

where τ_{net} is the net torque on the cylinder, I is the rotational inertia of the cylinder, and α is its angular acceleration. Calculate the individual torques, being careful about the signs, then sum them to obtain τ_{net}. Use $I = \frac{1}{2}MR_2^2$ to find the rotational inertia of the cylinder (see Table 11–2).

You should get $\tau_1 = 0.72\,\text{N}\cdot\text{m}$ (counterclockwise), $\tau_2 = 0.48\,\text{N}\cdot\text{m}$ (clockwise), $\tau_3 = 0.10\,\text{N}\cdot\text{m}$ (clockwise), and $\tau_4 = 0$. You should also get $I = 1.44 \times 10^{-2}\,\text{kg}\cdot\text{m}^2$.

(a) Consider the system consisting of the block, the disk, and Earth. Take the potential energy to be zero when the block is at the bottom of the 50 cm fall. Then the initial potential energy is $U_i = mgh$, where h is the distance fallen and m is the mass of the block. The initial kinetic energy is $K_i = 0$, since the block and disk are initially at rest. Suppose that after the block has fallen a distance h it has speed v and the disk has angular speed ω. Then the final kinetic energy is $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Conservation of mechanical energy yields

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\,,$$

where I is the rotational inertia of the disk. Since the string does not slip on the disk, $v = R\omega$. Substitute $\omega = v/R$ into the conservation of mechanical energy equation. Also substitute $I = \frac{1}{2}MR^2$. Solve for v. You should get

$$v = \sqrt{\frac{4mgh}{M + 2m}}\,.$$

(b) Look at the algebraic expression you derived for v. It does not contain R.