Exercises from Day 1 talk by Jeremy Rouse

1. Suppose that Q is a positive-definite quadratic form. Assume that Q represents 2, and Q also represents 3. Show that Q also represents 818.

2. Let $S = \mathbb{N}$ be the set of positive integers. Show that there is no positive-definite S-universal ternary quadratic form.

3. Let p be a prime number and $Q(x, y, z, w) = x^2 + py^2 + pz^2 + pw^2$.

(a) Show that if $p \not\equiv 1 \pmod 8$, then every positive integer n which is congruent to a square mod p and $n > p(4p - 5)$ is represented by Q.

(b) Show that if $p \equiv 3 \pmod 8$, then $n = p(4p - 5)$ is not represented by Q.