February 5, 2000

PHY 712 – Problem Set # 8

Read Chapter 3 of **Jackson**.

1. Convince yourself that Eqs. 3.62 and 3.70 are correct by expanding the expressions to second order. That is, verify the following:

$$P_l(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}^*(\hat{\mathbf{r}}) Y_{lm}(\hat{\mathbf{r}}') \tag{1}$$

and

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{lm}^*(\hat{\mathbf{r}}) Y_{lm}(\hat{\mathbf{r}}'). \tag{2}$$

2. Consider a charge distribution of the form:

$$\rho(r, \theta, \phi) = \rho_0 r^2 e^{-\alpha r} \cos^2(\theta), \tag{3}$$

where ρ_0 and α are constants.

   (a) Express $\rho(r, \theta, \phi)$ as a sum of radial functions time spherical harmonic functions in the form:

$$\rho(r, \theta, \phi) = \sum_{lm} \rho_{lm}(r)\, Y_{lm}(\theta, \phi). \tag{4}$$

   (b) Using the results of the previous problem, find the corresponding electrostatic potential which vanishes at ∞.