

Computational and experimental investigation of $\text{Na}_4\text{P}_2\text{S}_6$ as a promising solid electrolyte material for sodium metal batteries

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Motivation

- Kuhn et al.¹ observed that $\text{Na}_4\text{P}_2\text{S}_6$ crystallizes to form monoclinic space group **C2/m (#12)**
- Computational results of Rush et al.²: **Kuhn structure is meta-stable**
- Recent experimental results of Hood et al.³ also find the **C2/m** structure



Reexamine previous calculations to understand the stability of the Kuhn structure

- Theoretically, $\text{Na}_4\text{P}_2\text{S}_6$ and $\text{Li}_4\text{P}_2\text{S}_6$ are chemically and structurally related
- Using combined approach of NMR and X-ray, the new experimental analysis⁴ on $\text{Li}_4\text{P}_2\text{S}_6$ concludes the structure to be ordered with space group **P321 (#150)**



Property similarities (discrepancies) between $\text{Na}_4\text{P}_2\text{S}_6$ and $\text{Li}_4\text{P}_2\text{S}_6$

- Compared to $\text{Li}_4\text{P}_2\text{S}_6$, $\text{Na}_4\text{P}_2\text{S}_6$ is a competitive electrolyte candidate³



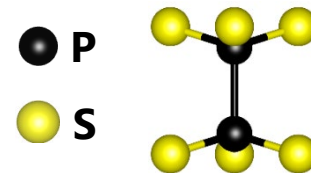
Conductivity mechanism

¹Kuhn et al., *Z. Anorg. Allg. Chem.* 640, 689-692 (2014).

²Rush et al., *Solid State Phys.* 286, 45-50 (2016).

³Hood et al., Manuscript in preparation.

⁴Neuberger et al., *Dalton Trans.* 47, 11691-11695 (2018).



$(\text{P}_2\text{S}_6)^{4-}$ with D_{3d} symmetry

Computational Methods

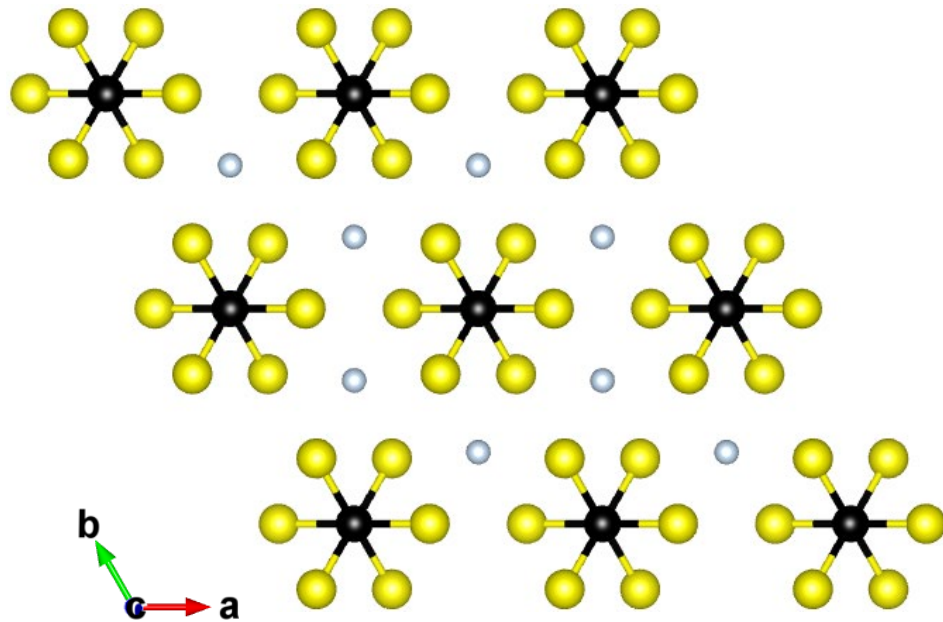
- ❑ Density Functional Theory (DFT) and Density Functional Perturbation Theory (DFPT) with the modified Perdew-Burke-Ernzerhof generalized gradient approximation (**PBEsol GGA**)
PRB 79 1551107 (2019)
- ❑ The projector augmented wave (PAW) formalism using ABINIT (<https://www.abinit.org>) & Quantum ESPRESSO (<http://www.quantum-espresso.org>)
- ❑ Datasets generated by ATOMPAW code available at <http://pwpaw.wfu.edu>
- ❑ Visualization software: XCrySDen, VESTA
- ❑ Space-group analysis: FINDSYM
- ❑ X-ray powder diffraction: Mercury

Previously reported results¹ obtained using (Local-density approximation) LDA

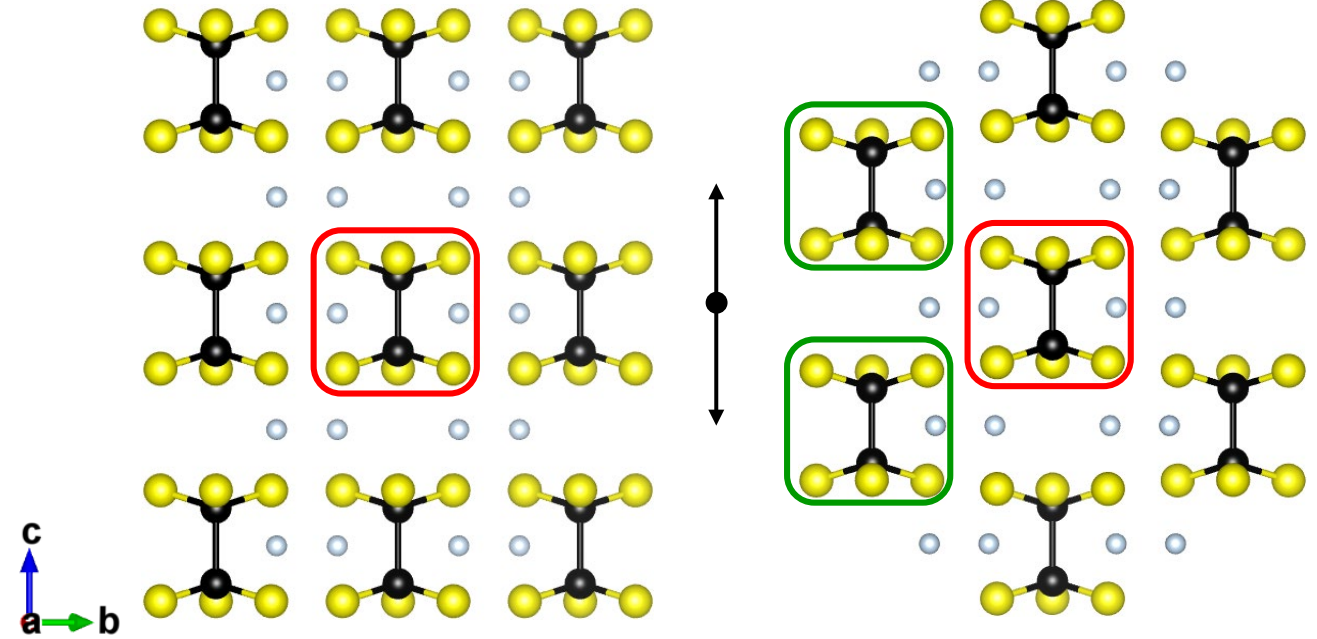
¹Rush et al., *Solid State Phys.* 286, 45-50 (2016).

Structure Analysis

● P ● S



Projection of the basic structure



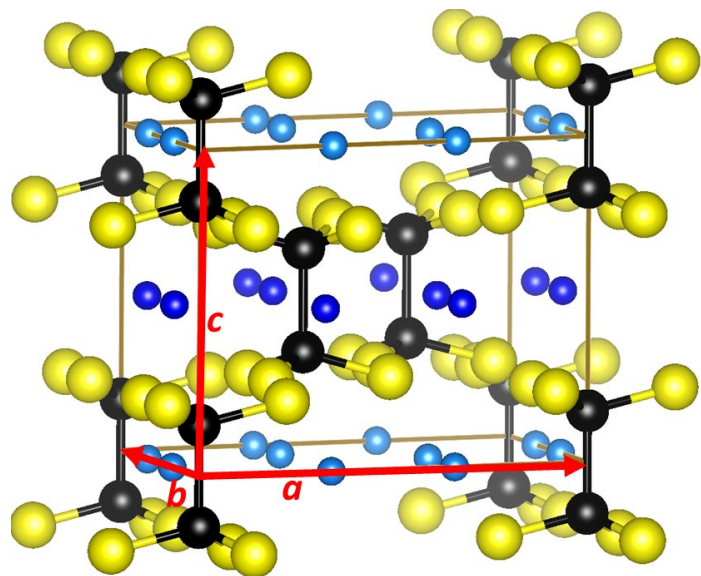
$$\mathbf{P}_{\uparrow} = \pm Z_P \vec{C}$$

$$\mathbf{P}_{\downarrow} = \pm \left(\frac{1}{2} - Z_P \right) \vec{C}$$

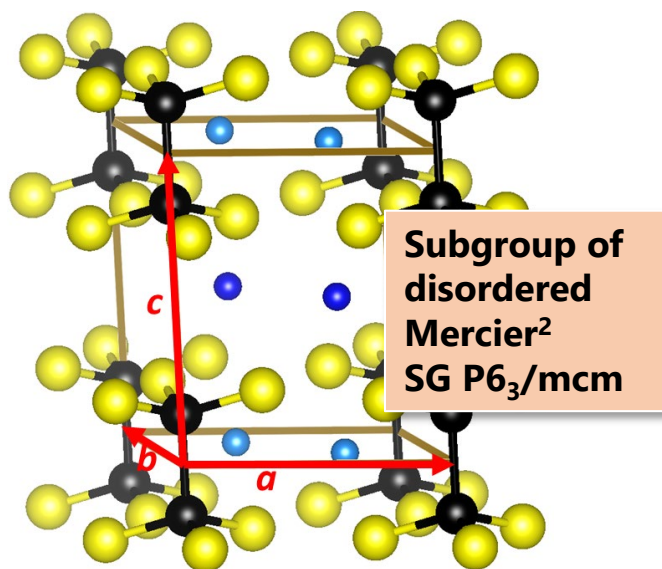
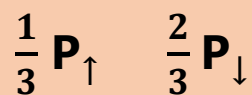
Hood et al., *J. Solid State Ionics* 284, 61 (2016).

Model Structures Considered

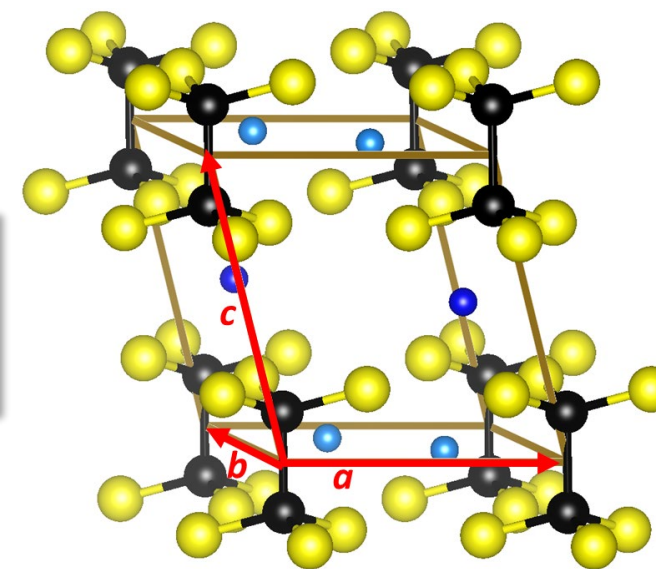
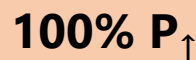
●● Inequiv. Na(Li) ● P ● S



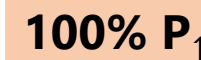
Hexagonal **P321** (#150)¹
3 formula units / unit cell



Hexagonal **P $\bar{3}$ 1m** (#162)³
1 formula unit / primitive unit cell



Monoclinic **C2/m** (#12)⁴
1 formula unit / primitive unit cell



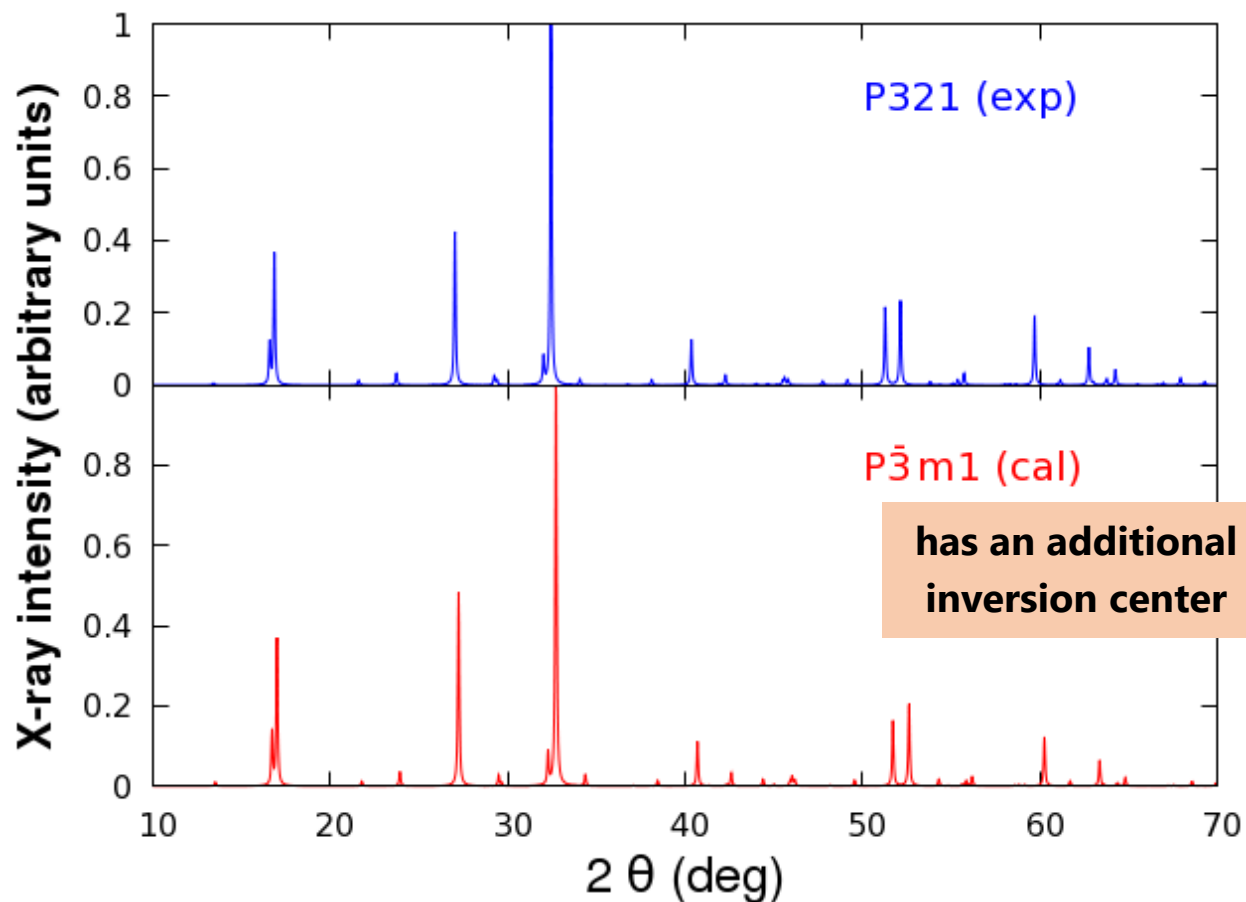
¹Neuberger et al., *Dalton Trans.* 47, 11691-11695 (2018).

²Mercier et al., *J. Solid State Chem.* 43, 151-162 (1982).

³Hood et al., *J. Solid State Ionics*, 284, 61 (2016).

⁴Kuhn et al., *Z. Anorg. Allg. Chem.* 640, 689-692 (2014).

More about the P321 Structure



Comparing X-ray diffraction patterns at $\lambda = 1.54056 \text{ \AA}$

Comparison of the fractional coordinates of $\text{Li}_4\text{P}_2\text{S}_6$ and $\text{Na}_4\text{P}_2\text{S}_6$ based on the Neuberger structure¹.

$\text{Li}_4\text{P}_2\text{S}_6$		Calculated			Experiment			
Atom	Wyck	x	y	z	Wyck	x	y	z
Li	6 g	0.666	0.000	0.000	3 e	0.625/0.683	0.000	0.0000
Li	6 h	0.667	0.000	$\frac{1}{2}$	3 f	0.631/0.671	0.000	$\frac{1}{2}$
P	2 c	0.000	0.000	0.171	2 c	0.000	0.000	0.170
P	2 d	$\frac{1}{3}$	$\frac{2}{3}$	0.663	2 d	$\frac{1}{3}$	$\frac{2}{3}$	0.668
P	2 d	$\frac{1}{3}$	$\frac{2}{3}$	0.324	2 d	$\frac{1}{3}$	$\frac{2}{3}$	0.335
S	6 i	0.110	0.220	0.242	6 g	0.108	0.217	0.241
S	6 i	0.114	0.557	0.254	6 g	0.122	0.561	0.250
S	6 i	0.447	0.224	0.259	6 g	0.452	0.226	0.255

$\text{Na}_4\text{P}_2\text{S}_6$		Calculated		
Atom	Wyck	x	y	z
Na	6 g	0.659	0.000	0.000
Na	6 h	0.676	0.000	$\frac{1}{2}$
P	2 c	0.000	0.000	0.157
P	2 d	$\frac{1}{3}$	$\frac{2}{3}$	0.660
P	2 d	$\frac{1}{3}$	$\frac{2}{3}$	0.342
S	6 i	0.102	0.205	0.229
S	6 i	0.129	0.564	0.271
S	6 i	0.463	0.231	0.264

¹Neuberger et al. *Dalton Trans.* 47, 11691-11695 (2018).

Static Lattice Results

Summary of static lattice results calculated with PBEsol GGA formalism. Lattice constants for the primitive unit cells are listed in units of Å and angles in degrees. The energies ΔE are listed as eV/(formula unit) referenced to the energy of the $P\bar{3}m1$ structure.

$\text{Li}_4\text{P}_2\text{S}_6$	a	b	c	α	β	γ	ΔE
$P\bar{3}1m$ (#162)	6.03	6.03	6.48	90.0	90.0	120.0	0.04
$C2/m$ (#12)	6.08	6.08	6.89	97.9	97.9	119.1	0.31
$P\bar{3}m1$ (#164) ^a	10.42	10.42	6.54	90.0	90.0	120.0	0.00
$\text{Na}_4\text{P}_2\text{S}_6$	a	b	c	α	β	γ	ΔE
$P\bar{3}1m$ (#162)	6.45	6.45	7.13	90.0	90.0	120.0	0.09
$C2/m$ (#12) ^b	6.51	6.51	7.52	98.5	98.5	117.6	0.00
$P\bar{3}m1$ (#164)	11.10	11.10	7.25	90.0	90.0	120.0	0.00

LDA results suggest that $C2/m$ structure is meta-stable

↑
static and vibrational →

^a Corresponding experimental values quoted from Neuberger et al., *Dalton Trans.* 47, 11691-11695 (2018) are $a = b = 10.51$ Å, $c = 6.59$ Å

^b Corresponding experimental values deduced from Kuhn et al., *Z. Anorg. Allg. Chem.* 640, 689-692 (2014) are $a = b = 6.54$ Å, $c = 7.54$ Å, $\alpha = \beta = 98.7$ deg, $\gamma = 118.1$ deg.

Phonon Spectrum of Na₄P₂S₆ in the C2/m Structure

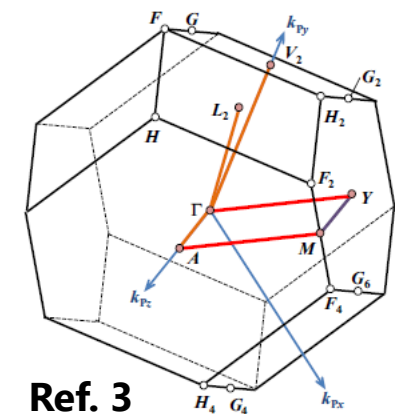
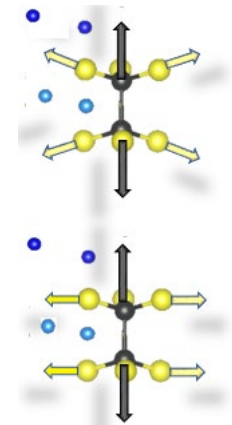
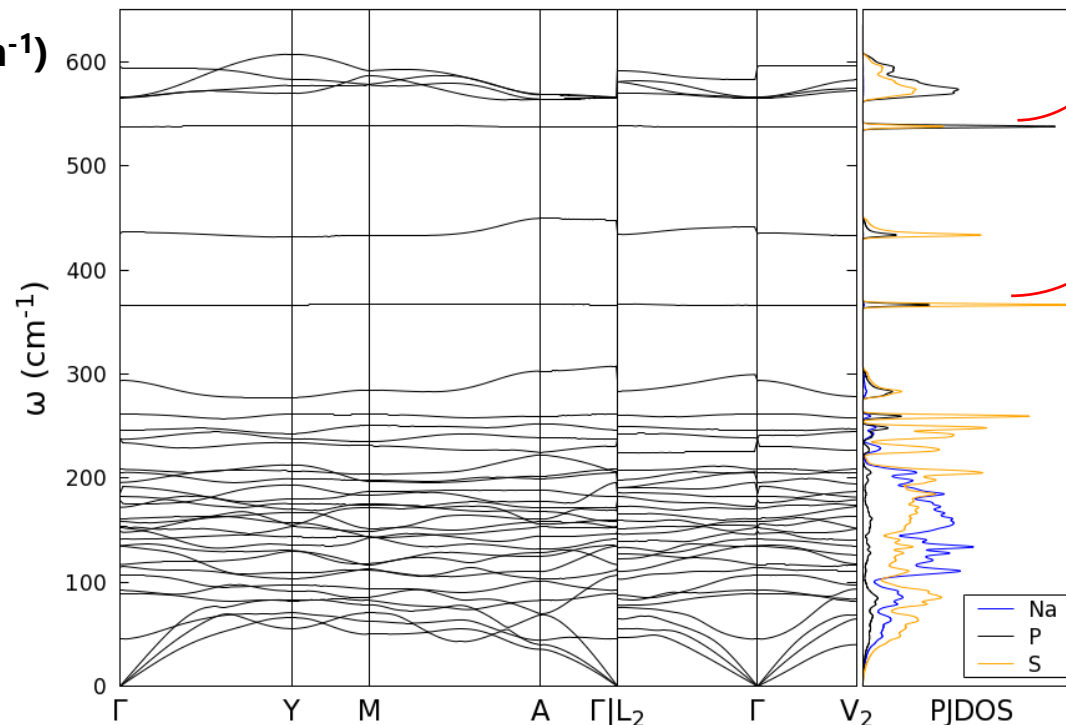


Density Functional Perturbation Theory (DFPT)¹: $(H_{SCF} - \varepsilon_n) |\Delta\psi_n\rangle = -(\Delta V_{SCF} - \Delta\varepsilon_n) |\psi_n\rangle$

$$\sum_{t,\beta} \left(\tilde{D}_{st}^{\alpha\beta}(\vec{q}) - \omega^2(\vec{q}) \delta_{st} \delta_{\alpha\beta} \right) u_t^\beta(\vec{q}) = 0 \quad \longrightarrow \quad \text{Eigenvalues and Eigenvectors } (\omega(q), \{u_i\})$$

Comparisons of main Raman active modes (cm⁻¹)

Cal.	Exp. ²
149	152
196	203
261	273
366	383
537	560
565	577



Ref. 3
mC ($b > a \sin \beta$ elongated dodecahedron):
 Γ -Y-M-A- Γ | L_2 - Γ - V_2

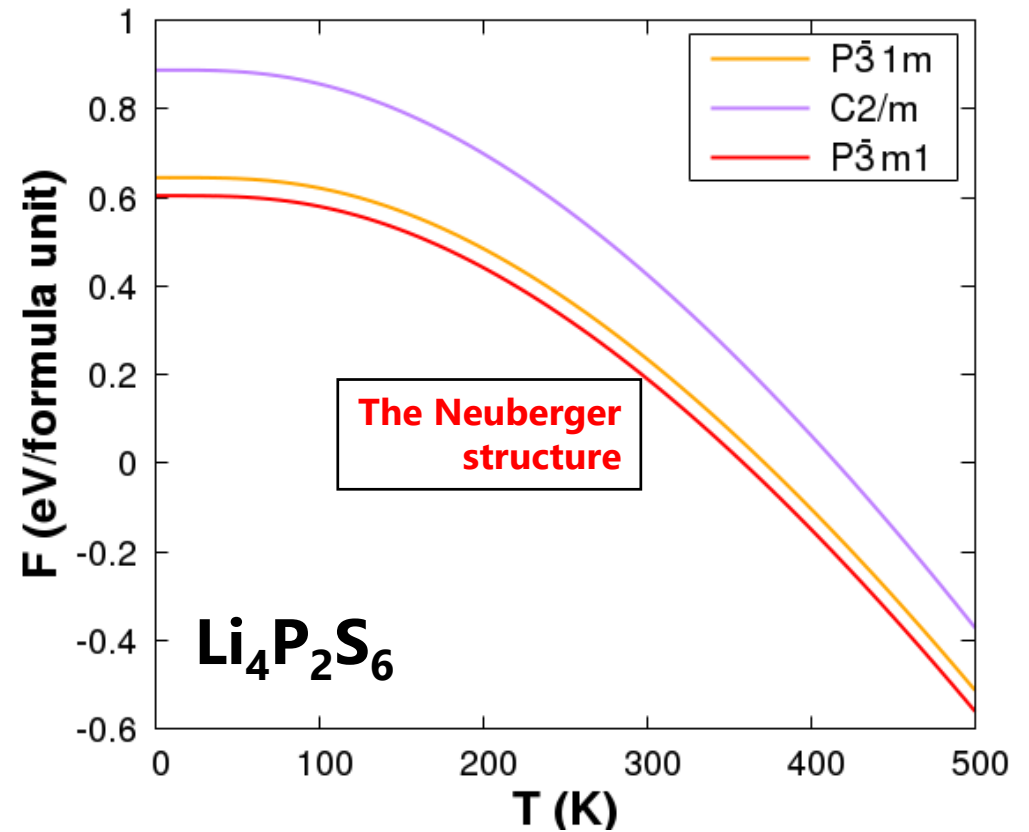
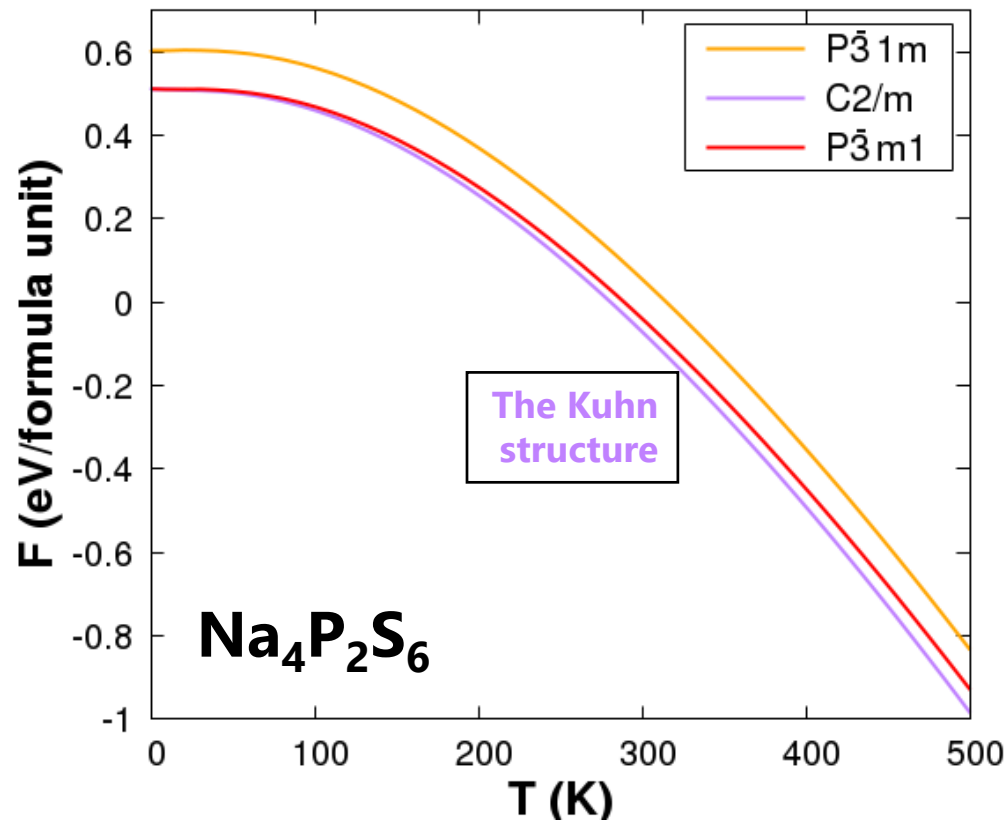
12 atoms/cell × 3 dimensions
→ 36 phonons (vibrational modes)

¹Baroni et al., *Rev. Mod. Phys.* 73, 515-562 (2001).
²Hood et al., Manuscript in preparation.
³Hinuma et al., *Comp. Mat. Sci.* 128, 140-184 (2017).

Stability Analysis

Helmholtz free energy: $\mathbf{F} = \mathbf{U}_{\text{SL}} + \mathbf{F}_{\text{vib}}$

Where U_{SL} is the static lattice internal energy and F_{vib} is the phonon free energy in harmonic approximation



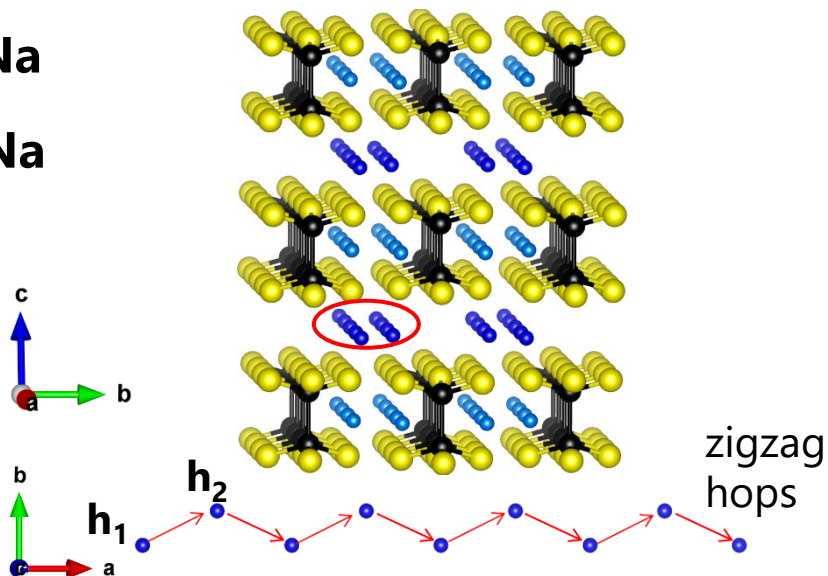
Ion Migration of $\text{Na}_4\text{P}_2\text{S}_6$ in the C2/m Structure

● *g* site Na

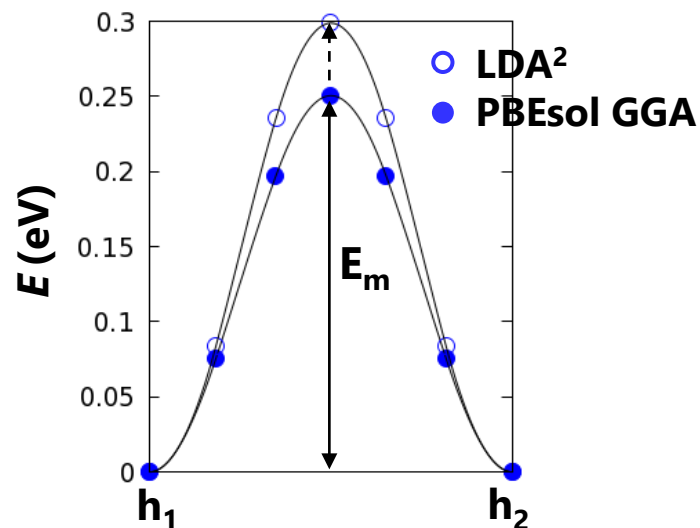
● *h* site Na

● P

● S



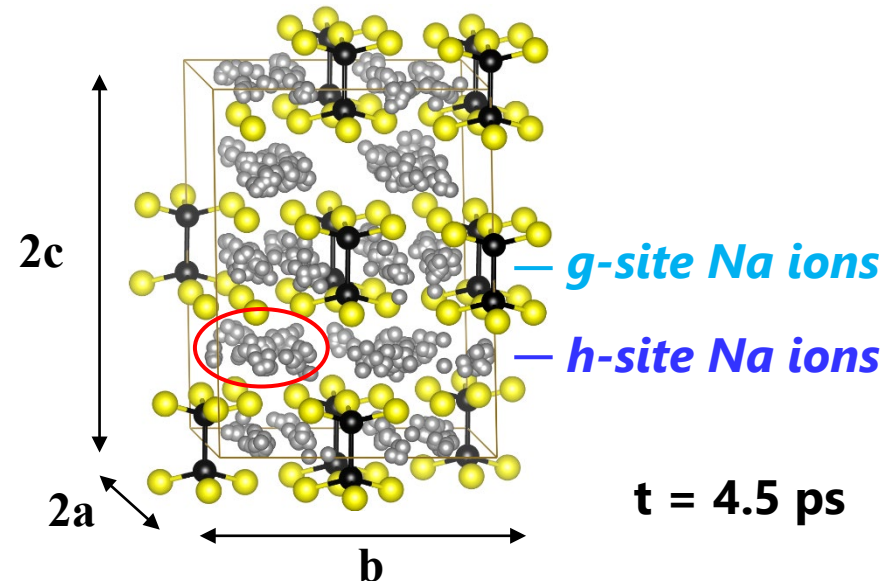
Nudged
Elastic
Band
Method¹



$$E_A^{NEB} = E_m + \frac{1}{2}E_f, \quad \sigma(T) = \frac{K}{T} e^{-E_A/k_B T}$$

Comparing activation energies E_A (eV) for Na ion conductivity

PBEsol GGA	LDA ²	Exp ³
0.34	0.42	0.39



Ab-initio Molecular Dynamics with $T = 1000 \text{ K}$

¹Henkleman et al., *J. Chem. Phys.* 113, 9901-9904 (2000).

²Rush et al., *Solid State Phys.* 286, 45-50 (2016).

³Hood et al., Manuscript in preparation.

Summary

- ❑ According to PBEsol GGA results, $\text{Na}_4\text{P}_2\text{S}_6$ is to be stabilized in the C2/m structure and $\text{Li}_4\text{P}_2\text{S}_6$ is to be stabilized in the $\text{P}\bar{3}\text{m}1$ structure
- ❑ PBEsol GGA and LDA results of activation energy for Na ion migration reasonably agree with the experimental measurements which suggest a viable solid electrolyte
- ❑ MD simulations are expected to provide more information on understanding the conductivity mechanisms
- ❑ Calculations will be performed to investigate the stability of $\text{Na}_4\text{P}_2\text{S}_6/\text{Na}$ interface

Acknowledgements

- ❑ This work was supported by NSF grant DMR-1507942
- ❑ Computations were performed on the Wake Forest University DEAC cluster, a centrally managed resource with support provided in part by the University
- ❑ Thank my advisor Prof. Natalie A. W. Holzwarth and our experimental collaborator Dr. Zachary D. Hood