

# PHY 711 – Assignment #7

September 8, 2006

In class, we considered how to describe the Lorentz force on a particle of charge  $q$  in an electric field  $\mathbf{E}$  and magnetic field  $\mathbf{B}$ :

$$\mathbf{F}_{\text{EM}} = \frac{q}{c} \mathbf{v} \times \mathbf{B} + q\mathbf{E} \quad (1)$$

in the Lagrangian formulation.

Assuming we can describe these electric and magnetic fields in terms of the scalar and vector potentials according to

$$\mathbf{E} = -\nabla\phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad (2)$$

and

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (3)$$

the electromagnetic contribution to the Lagrangian is

$$L_{\text{EM}} = \frac{q}{c} \dot{\mathbf{r}} \cdot \mathbf{A} - q\phi. \quad (4)$$

1. Show that

$$\mathbf{F}_{\text{EM}}|_z = \frac{\partial L_{\text{EM}}}{\partial z} - \frac{d}{dt} \frac{\partial L_{\text{EM}}}{\partial \dot{z}}. \quad (5)$$

2. Consider what happens to the Lagrangian and to the Lorentz force when the vector and scalar potentials are changed according to

$$\mathbf{A}' = \mathbf{A} + \nabla\psi, \quad (6)$$

and

$$\phi' = \phi - \frac{\partial \psi}{\partial t}. \quad (7)$$

Here, the scalar function  $\psi(x, y, z, t)$  is arbitrary.