September 1, 2005

PHY 711 – Problem Set # 5

Continue reading Chapter 3 of **Fetter & Walecka**.

1. Suppose you want to extremize an integral of the form:

$$\mathcal{I} \equiv \int_{x_1}^{x_2} \phi(y, y', y''; x)\, dx,$$

where $y = y(x)$, $y' \equiv \frac{dy}{dx}$, and $y'' \equiv \frac{d^2y}{d^2x}$, and where ϕ is a given function. Find the generalization of the Euler-Lagrange equation (Eqs. 17.19 & 34 in your text) for this function ϕ. In order to solve this problem, you should assume that the end point values and slopes are all fixed (not varying). That is, $y(x_1) = A$, $y(x_2) = B$, $y'(x_1) = C$, and $y'(x_2) = D$, where A, B, and C, and D are all fixed values.

2. *Extra Credit*

Consider an integral of the form:

$$E = \int d^3r\, F[\rho(\mathbf{r}), g(\mathbf{r})],$$

where $\rho(\mathbf{r})$ is a function of position in three dimensions, and $g(\mathbf{r}) \equiv |\nabla\rho(\mathbf{r})|$. Show that the functional variation of E with respect to $\rho(\mathbf{r})$ is given by:

$$\delta E = \int d^3r \left\{ \frac{\partial F}{\partial \rho} - \frac{\partial F}{\partial g}\left(\frac{\nabla^2\rho}{g} - \frac{\nabla\rho\cdot\nabla g}{g^2} \right) - \frac{\partial^2 F}{\partial\rho\partial g} g - \frac{\partial^2 F}{\partial^2 g}\frac{\nabla\rho\cdot\nabla g}{g} \right\} \delta\rho(\mathbf{r}).$$