PHY 113 – Additional notes for Chapter 1 (Problem Set # 1)

In class, we did not quite have enough time to discuss error analysis. This will be discussed also in your laboratory work. Some (hopefully) helpful comments follow:

Some degree of error is associated with any measurement. For example, Suppose your ruler has centimeter and millimeter markings. If you measured one side of your text you could say that its length is $l_1 \pm \delta l_1$ (for example 22.2 ± 0.2) cm. Suppose the second length is measured as $l_2 \pm \delta l_2$, while the thickness is $t \pm \delta t$. If you now wanted to compute the volume of your text, that would be

$$V = l_1 \cdot l_2 \cdot t.$$

To get an idea of the error in your calculation you need to think about the error in each length measurement. Precisely,

$$\delta V \equiv (l_1 \pm \delta l_1)(l_2 \pm \delta l_2)(t \pm \delta t) - l_1 l_2 t.$$

If we want an estimate of the error, then we can make the following approximations to the above formula for δV.

δl_1 is small so that terms like $\delta l_1 \cdot \delta l_2$ and $\delta l_1 \cdot \delta l_2 \cdot \delta t$ can be neglected.

Since we want to estimate the maximum possible error, we should replace $\pm$ with $+$.

Therefore,

$$\delta V \approx \delta l_1 l_2 t + l_1 \delta l_2 t + l_1 l_2 \delta t.$$

If we divide this result by V, we get the very compact result:

$$\frac{\delta V}{V} = \frac{\delta l_1}{l_1} + \frac{\delta l_2}{l_2} + \frac{\delta t}{t}.$$

This shows that in this case the fractional error is equal to the sum of the fractional errors in each of the length measurements. Not all derived quantities will have this simple result, but often one can estimate the error as a function of fractional errors. Homework problem # 4 makes use of some of these ideas.