Physics 741 – Graduate Quantum Mechanics 1

Solutions to Chapter 11

1. [5] An electron in an unknown spin state $|a\rangle$ is brought into proximity with a second electron in a known spin state $|b\rangle$. We wish to make the spin of the second electron match the first. A *quantum Xerox device* will copy it onto the second spin, so $U_{\text{Xerox}}|a,b\rangle = |a,a\rangle$. A *quantum teleporter* will swap the two spin states, as $U_{\text{Teleport}}|a,b\rangle = |b,a\rangle$.

(a) [3] By considering the three initial spin states $|a\rangle = |+\rangle$, $|a\rangle = |-\rangle$, and $|a\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$, show that the quantum Xerox device is impossible.

If the quantum Xerox device exists, it must change the state $|+,b\rangle$ into $|+,+\rangle$ and $|-,b\rangle$ into $|-,-\rangle$, in other words

$$U_{\text{Xerox}}|+,b\rangle = |+,+\rangle \quad \text{and} \quad U_{\text{Xerox}}|-,b\rangle = |-,-\rangle$$

However, U_{Xerox} is a linear operator, and it follows that

$$U_{\text{Xerox}}\left[\tfrac{1}{\sqrt{2}}\left(|+,b\rangle + |-,b\rangle\right)\right] = \tfrac{1}{\sqrt{2}}\left(|+,+\rangle + |-,-\rangle\right).$$

However, the quantum Xerox device is supposed to evolve this state into

$$U_{\text{Xerox}}\left[\tfrac{1}{\sqrt{2}}\left(|+,b\rangle + |-,b\rangle\right)\right] = \tfrac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) \otimes \tfrac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) = \tfrac{1}{2}\left(|++\rangle + |+-\rangle + |-+\rangle + |--\rangle\right)$$

Obviously, these equations are inconsistent, and hence this is impossible.

(b) [2] By considering the same three initial states, show that the same problem does not apparently occur for the quantum teleport device.

The quantum teleport device should evolve the states according to

$$U_{\text{Teleport}}|+,b\rangle = |b,+\rangle \quad \text{and} \quad U_{\text{Teleport}}|-,b\rangle = |b,-\rangle$$

and therefore by linearity,

$$U_{\text{Teleport}}\left[\tfrac{1}{\sqrt{2}}\left(|+,b\rangle + |-,b\rangle\right)\right] = \tfrac{1}{\sqrt{2}}\left(|b,+\rangle + |b,-\rangle\right)$$

But this is exactly what we would want it to do, so there is, in fact, no problem in this case.