Physics 741 – Graduate Quantum Mechanics 1

# Solutions to Chapter 10

**4. [10] Look up and/or calculate the number density of atoms in copper. Assume there is one conduction electron per atom.**

**(a) [6] What is the Fermi energy (in eV) and Fermi degeneracy pressure (in GPa) for electrons at this density?**

Using Wikipedia, we can find the atomic mass of copper and the density, which together with Avogadro's number tells us the number density of atoms.

$$n = \frac{\rho}{m} = \frac{8.96\ \text{g/cm}^3}{63.546\ \text{g/mol}} 6.022\times10^{23}\ \text{mol}^{-1} = 8.49\times10^{22}\ \text{cm}^{-3} = 8.49\times10^{28}\ \text{m}^{-3}$$

We now put this into our formula for the Fermi energy

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}} = \frac{\left(1.0546\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\cdot 9.1094\times10^{-31}\ \text{kg}}\left[3\pi^2\cdot 8.49\times10^{28}\ \text{m}^{-3}\right]^{\frac{2}{3}} = \frac{1.128\times10^{-18}\ \text{J}}{1.602\times10^{-19}\ \text{J/eV}} = 7.044\ \text{eV}$$

The degeneracy pressure is

$$P_F = \tfrac{2}{5} n E_F = \tfrac{2}{5}\left(1.128\times10^{-18}\ \text{J}\right)\left(8.49\times10^{28}\ \text{m}^{-3}\right) = 3.831\times10^{10}\ \text{N/m}^2 = 38.31\ \text{GPa}$$

**(b) [4] The bulk modulus is defined as $K = -V\left(\partial P/\partial V\right)$. Find a formula for the bulk modulus due to electron degeneracy pressure. Use this to estimate the bulk modulus of copper (in GPa). Look it up somewhere for copper and compare the result.**

Starting from the formula for degeneracy pressure, we have

$$K_F = -V\frac{\partial}{\partial V}\left[\frac{\hbar^2}{5m}\left(3\pi^2\right)^{\frac{2}{3}} N^{\frac{5}{3}} V^{-\frac{5}{3}}\right] = \frac{5}{3}\frac{\hbar^2}{5m}\left(3\pi^2\right)^{\frac{2}{3}} N^{\frac{5}{3}} V^{-\frac{5}{3}} = \tfrac{5}{3} P_F = 63.84\ \text{GPa}$$

The correct experimental value, according to Wikipedia, is 140 GPa, so this is about a factor of 2.2 too low. I must assume that other electrons besides the conduction electrons are contributing to the pressure.