Physics 741 – Graduate Quantum Mechanics 1

Homework Set B
Due Friday, September 5

1. For each of the normalized wave functions given below, find the Fourier transform $\tilde{\psi}(k)$, and check that it satisfies the normalization condition $\int_{-\infty}^{\infty} \left|\tilde{\psi}(k)\right|^2 dk = 1$

   (a) $\psi(x) = (A/\pi)^{1/4} \exp\left(iKx - \frac{1}{2}Ax^2\right)$

   (b) $\psi(x) = \sqrt{\alpha}\, \exp\left(-\alpha|x|\right)$

2. For each of the wave functions in question 1, find $\bar{x}$, Δx, $\bar{p}$, Δp, and check that the uncertainty relationship

   $$(\Delta x)(\Delta p) \geq \tfrac{1}{2}\hbar$$

   is satisfied.

3. A particle of mass *m* lies in the harmonic oscillator potential, given by

   $$V(x) = \tfrac{1}{2} m\omega^2 x^2$$

   Later we will solve this problem exactly, but for now, we only want an approximate solution.

   (a) Let the uncertainty in the position be $\Delta x = a$. What is the corresponding minimum uncertainty in the momentum Δp? Write an expression for the total energy (kinetic plus potential) as a function of *a*.

   (b) Find the minimum of the energy function you found in (a), and thereby estimate the minimum energy (called zero point energy) for a particle in a harmonic oscillator. Your answer should be very simple.