Physics 744 - Field Theory

Homework Set 4

1. A single real scalar field has the usual Lagrangain, $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$.
   (a) Work out the components of T^{0i}, three components of the stress-energy tensor. Write it in terms of $\phi(\mathbf{x})$, $\nabla\phi(\mathbf{x})$, and $\pi(\mathbf{x})$. Also, write an expression for $\vec{P}$, the three-momentum.
   (b) Substitute the expression for $\phi(\mathbf{x})$ and $\pi(\mathbf{x})$ in terms of creation and annihilation operators into this expression, and hence find a simple expression for $\vec{P}$ in terms of creation and annihilation operators $\alpha_{\vec{k}}^\dagger$ and $\alpha_{\vec{k}}$.
   (c) Consider the state $|\vec{p}\rangle = \alpha_{\vec{p}}^\dagger|0\rangle$. Show that it is an eigenstates of $\vec{P}$ and determine its eigenvalue.

2. A single scalar field has the usual Lagrangian $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$. It is in the state

$$|\psi\rangle = N\exp\left(z\alpha_{\vec{p}}^\dagger\right)|0\rangle = N\sum_{n=0}^{\infty}\frac{z^n\left(\alpha_{\vec{p}}^\dagger\right)^n}{n!}|0\rangle,$$

   where z is an arbitrary complex number, and N is a normalization constant.
   (a) Show that $\left[\alpha_{\vec{k}},\left(\alpha_{\vec{p}}^\dagger\right)^n\right] = n\left(\alpha_{\vec{p}}^\dagger\right)^{n-1}\left(2\omega_{\vec{p}}\right)\left(2\pi\right)^3\delta^3\left(\vec{k}-\vec{p}\right)$.
   (b) Show that $|\psi\rangle$ is an eigenstates of $\alpha_{\vec{k}}$ and determine its eigenvalue (it will be zero unless $\vec{k}=\vec{p}$).
   (c) Assume N is chosen so that $|\psi\rangle$ is normalized, that is, $\langle\psi|\psi\rangle = 1$. Evaluate $\langle\psi|\phi(\mathbf{x})|\psi\rangle$.