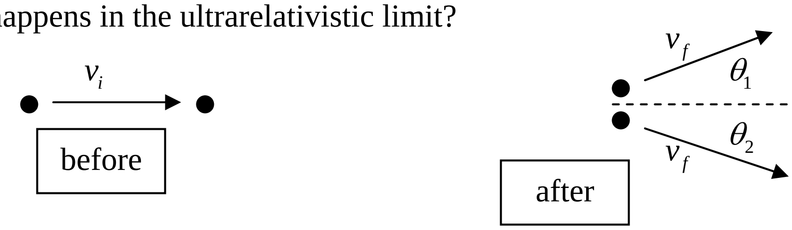
Physics 744 - Field Theory

Homework Set 2

1. Let **x**, **y, z**, and **w** be four independent four-vectors. We wish to form a scalar quantity s that is Lorentz invariant under proper Lorentz transformations and is linear in each of these four quantities, *i.e.*, it will contain expressions like *xyzw*, but we want to show explicitly how the indices can be put together.
   (a) What is the most general expression that can be formed of this type? There should be four linearly independent terms.
   (b) A term is called a *true scalar* if it is invariant under parity, and a *pseudoscalar* if it changes sign under parity. Classify the four terms as scalars or pseudoscalars.

2. In classical physics, if an object of mass m hits an object of identical mass, the two objects will head off at a 90 degree angle compared to each other. Consider an object of mass m moving at speed v_i and colliding elastically with another object of mass m. The two move off at identical speeds v_f at angles θ_1 and θ_2.
   (a) Write the four-momentum of all the incoming and outgoing particles, and write the conservation of four-momentum in components.
   (b) Show that $\theta_1 = \theta_2$.
   (c) Find a formula for γ_f in terms of the initial velocity.
   (d) Show that the final angle is given by $\cos^2\theta = (\gamma_i + 1)/(\gamma_i + 3)$. Hence show that the outgoing particles are perpendicular in the non-relativistic limit. What happens in the ultrarelativistic limit?



3. A Z-particle (mass m_Z) at rest decays to an electron (mass effectively zero) with energy E_1, a positron (also massless) with energy E_2 moving at an angle θ compared to it, and an invisible X particle of unknown mass. Find a formula for the unknown mass m_X^2