# Final Information

Equations you should memorize:

$$c = \hbar = \varepsilon_0 = \mu_0 = 1 . \tag{1.18}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}} = \frac{L_0}{L} = \frac{t}{\tau} = \frac{E}{m} , \tag{2.8}$$

$$u \cdot v \equiv g_{\alpha\beta} u^\alpha v^\beta = u^0 v^0 - \mathbf{u} \cdot \mathbf{v} . \tag{2.11}$$

$$m^2 \equiv p \cdot p = E^2 - \mathbf{p}^2 . \tag{2.33}$$

$$\mathbf{v} = \frac{\mathbf{p}}{E} . \tag{2.34}$$

$$s \equiv \left(p_1 + p_2\right)^2 = \left(E_1 + E_2\right)^2 - \left(\mathbf{p}_1 + \mathbf{p}_2\right)^2 . \tag{2.35}$$

$$p^\mu = \left(E, \mathbf{p}\right) = \left(E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta\right) .$$

$$\left(\bar{\Psi}_A \Gamma_1 \Gamma_2 \cdots \Gamma_n \Psi_B\right)^* = \bar{\Psi}_B \bar{\Gamma}_n \cdots \bar{\Gamma}_2 \bar{\Gamma}_1 \Psi_A . \tag{3.43}$$

$$\not{p} \equiv p_\mu \gamma^\mu . \tag{3.47}$$

$$\not{p}^2 = p^2$$

$$\gamma^\mu \gamma_5 = -\gamma_5 \gamma^\mu$$

$$\left| t_1, p_1, s_1 ; t_2, p_2, s_2 \right\rangle = \begin{cases} -\left| t_2, p_2, s_2 ; t_1, p_1, s_1 \right\rangle & \text{if two fermions ,} \\ \left| t_2, p_2, s_2 ; t_1, p_1, s_1 \right\rangle & \text{otherwise .} \end{cases} \tag{4.5}$$

$$\mathrm{Tr}\left(\gamma^{\mu_1}\gamma^{\mu_2} \cdots \gamma^{\mu_{2N+1}}\right) = \mathrm{Tr}\left(\gamma_5 \gamma^{\mu_1}\gamma^{\mu_2} \cdots \gamma^{\mu_{2N+1}}\right) = 0 . \tag{6.1}$$

$$\mathrm{Tr}\left(1\right) = 4 , \tag{6.2a}$$

$$\mathrm{Tr}\left(\gamma^\mu \gamma^\nu\right) = 4 g^{\mu\nu} , \tag{6.2b}$$

$$\mathrm{Tr}\left(\gamma_5\right) = \mathrm{Tr}\left(\gamma_5 \gamma^\alpha \gamma^\beta\right) = 0 , \tag{6.3a}$$

Feynman Diagram things you should know:

- How to draw Feynman diagrams
- How to get the amplitudes from them:
  - Propagator for scalar, fermion and photon/gluon (not *W* or *Z* propagator):
  - Follow fermion lines backwards from head to tail
  - Subtract diagrams with swapped fermion lines, otherwise add
- How to square them
  - Sum on final spins – average over initial spins
  - Turn them into traces for fermions
- How to get differential/total cross-sections and decay rates
  - Factor of 1/*n*! for identical final particles in total (not differential)

## The standard model

Particles: You should memorize:

- Names, abbreviations, charges, spins, and number of colors for all standard model particles
- Which generations the fermions belong to
  - $1^{st}$ (lightest) generation: u,d,e,$\nu_e$
  - $2^{nd}$ (medium) generation: c,s,$\mu$,$\nu_\mu$
  - $3^{rd}$ (heavy) generation: t,b,$\tau$,$\nu_\tau$
- Which particles have masses (quarks, charged leptons, W, Z, Higgs)
- Which quarks are involved with $SU(3)_F$ symmetry: u, d, s
- Which gauge bosons are responsible for each force:
  - photon: QED
  - gluon: QCD
  - *W* and *Z*: weak forces

For QED:

- Memorize the fermion-photon Feynman rule, the photon and fermion propagators (Fig. 7-2)

For Strong Forces:

- Understand how to use raising and lowering operators $I_\pm$ for isospin states
  - On both kets and bras
- Understand how to use $T_{i\to j}$ and states like $\left|B^*_{ijk}\right\rangle$ and $\left|M_i^{\,j}\right\rangle$
- Understand that isospin generators (approximately) and SU(3) operators (very approximately) commute with the Hamiltonian density $\mathcal{H}$.

For QCD:

- Which particles have color, and hence strong interactions
- Memorize the drawings for the quark-gluon coupling and the gluon self-couplings. You don't need to know the Feynman rule

For Weak interactions:

- Which *W*-couplings are allowed for fermions
  - In leptons, connect charged lepton with its corresponding neutrino
  - In quarks, connect any up-type quark with any down-type quark
  - But CKM contribution is large only when you stay within a generation
- Which Z-couplings are allowed for fermions
  - Every fermion couples only to itself

For the Higgs and standard model

- Higgs is responsible for all masses (quarks, charged leptons, *W* and *Z*)
- The potential of the Higgs field is not at zero, but at some non-zero value
  - This breaks the symmetry, allowing non-gauge invariant effects
- The gauge group of the standard model is SU(3)xSU(2)xU(1)
  - SU(3) is strong, U(1) lives in the SU(2)xU(1), and the weak interactions are what are left over in SU(2)xU(1)

You will be provided with:

- Everything, or nearly everything, on page v (penultimate page of book)
- All of the equations on the next page

# Useful Formulas and Identities

## Units and Constants

$1\ \text{eV} = 1.602\times10^{-19}\ \text{J}$

$6.582\times10^{-16}\ \text{s}\cdot\text{eV} = 1$

$1\ \text{s} = 3\times10^{8}\ \text{m}$

$1\ \text{kg} = 5.6\times10^{26}\ \text{GeV}$

$197\,\text{MeV}\cdot\text{fm} = 1$

$1\text{b} = 100\ \text{fm}^2$

$m_e = 0.51100\ \text{MeV}$

$m_p = 938.27\ \text{MeV}$

## Metric Prefixes

| | |
|---|---|
| T | $10^{12}$ |
| G | $10^{9}$ |
| M | $10^{6}$ |
| k | $10^{3}$ |
| m | $10^{-3}$ |
| μ | $10^{-6}$ |
| n | $10^{-9}$ |
| p | $10^{-12}$ |
| f | $10^{-15}$ |

## Dirac Properties

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\{\gamma^\mu, \gamma_5\} = 0$$

$$\gamma_5\gamma_5 = 1$$

$$\bar{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0$$

$$\bar{\gamma}^\mu \equiv \gamma^\mu$$

$$\bar{\gamma}_5 = -\gamma_5$$

## Spinors

$$\not{p}u = mu$$

$$\bar{u}\not{p} = \bar{u}m$$

$$\not{p}v = -mv$$

$$\bar{v}\not{p} = -\bar{v}m$$

$$\sum_s u\bar{u} = \not{p} + m$$

$$\sum_s v\bar{v} = \not{p} - m$$

## Dirac Trace Identities

$$\text{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = 4\left(g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\alpha}g^{\nu\beta}\right)$$

$$\text{Tr}\left(\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = -4i\varepsilon^{\mu\nu\alpha\beta}$$

## Cross-Sections and Decay Rates

$$\Gamma = \frac{D}{2M} \qquad \sigma = \frac{D}{4\left|E_2\mathbf{p}_1 - E_1\mathbf{p}_2\right|}$$

$$D(\text{two}) = \frac{p}{16\pi^2 E_{\text{cm}}}\int\left|i\mathcal{M}\right|^2\, d\Omega$$

$$D(\text{three}) = \frac{1}{8(2\pi)^5}\int dE_1\, dE_2\, d\Omega_1\, d\phi_{12}\left|i\mathcal{M}\right|^2$$

## Luminosity

$$L = f\,\frac{nN_1N_2}{A}.$$

## Spinors and Polarizations

$$\not{p}\,u(p,s) = mu(p,s)$$

$$\not{p}\,v(p,s) = -mv(p,s)$$

$$\sum_s u(p,s)\bar{u}(p,s) = \not{p} + m$$

$$\sum_s v(p,s)\bar{v}(p,s) = \not{p} - m$$

$$q\cdot\varepsilon(q,\lambda) = 0$$

$$\varepsilon^*(q,\lambda)\cdot\varepsilon(q,\tau) = \delta_{\lambda\tau}$$

massless:

$$\sum_\lambda \varepsilon^{*\mu}(q,\lambda)\varepsilon^\nu(q,\lambda) = -g^{\mu\nu}$$

massive:

$$\sum_\lambda \varepsilon^{*\mu}(q,\lambda)\varepsilon^\nu(q,\lambda) = -g^{\mu\nu} + q^\mu q^\nu / M^2$$