

Solution Set 8

Let's first figure out which of the ones listed correspond to which ones in the table for the group O . Obviously, E is item (1) on Natalie's list. C_4 means a rotation around the principle axis by 90 degrees, which has the property that it exchanges two coordinates and places a minus sign on one of them (items 10, 11, 19, 20, 26, 28). For $C_2 = C_4^2$, we must mean 180 degree rotation around one of these same axes, which reverses two of the coordinates while leaving the third alone (5, 6, 7). For the other C_2 , we must mean rotating around a diagonal going from one edge of the cube to the opposite edge, for example, the $x + y$ axis or the $x - y$ axis. The former swaps x and y , while also reversing z , while the latter both swaps x and y and throws in a minus sign, while reversing z . Hence we are looking for items where two coordinates have been swapped, the unswapped one is negative, and the other two are either both plus or both minus (12, 16, 18, 24, 27, 32). This leaves the C_3 rotations, which means rotating around one of the long diagonals. Such a rotation always produces a three-fold permutation of the axes, for example, if you rotate around the axis $x + y + z$, you get something like $(x, y, z) \rightarrow (y, z, x)$. However, if you rotate around some other axis, like $x + y - z$, you get something like $(x, y, z) \rightarrow (-y, z, -x)$, so we are looking for cyclic permutations with even numbers of sign changes (33, 37, 38, 39, 41, 45, 46, 47).

To categorize the other twenty-four, we just find the matching ones with all the signs reversed, which represents inversion. I then named the categories $iE = i$, $iC_4 = S_4$, $iC_4^2 = \sigma_v$, $iC_2 = \sigma_d$, and $iC_3 = S_6$.

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|------------------|----------------|----------------|----------------|-----------|-----------|
| 1. E | 9. σ_d | 17. σ_d | 25. σ_d | 33. C_3 | 41. C_3 |
| 2. σ_v | 10. C_4 | 18. C_2 | 26. C_4 | 34. S_6 | 42. S_6 |
| 3. σ_v | 11. C_4 | 19. C_4 | 27. C_2 | 35. S_6 | 43. S_6 |
| 4. σ_v | 12. C_2 | 20. C_4 | 28. C_4 | 36. S_6 | 44. S_6 |
| 5. $C_4^2 = C_2$ | 13. σ_d | 21. S_4 | 29. S_4 | 37. C_3 | 45. C_3 |
| 6. $C_4^2 = C_2$ | 14. S_4 | 22. S_4 | 30. σ_d | 38. C_3 | 46. C_3 |
| 7. $C_4^2 = C_2$ | 15. S_4 | 23. σ_d | 31. S_4 | 39. C_3 | 47. C_3 |
| 8. i | 16. C_2 | 24. C_2 | 32. C_2 | 40. S_6 | 48. S_6 |