Physics 745 - Group Theory

Homework Set 28

Due Friday, April 17

1. The group SU(3) contains the group SU(2) as a subgroup, and in more than one way
   (a) Show that the generators T_1, T_2 and T_3 form an SU(2) subgroup; that is, show that $[T_1, T_2] = iT_3$, etc. To save time, only do two of the three commutators. How does the 3 representation of SU(3) break into representations under this subgroup?
   (b) Show that the generators $2T_2$, $2T_5$, $2T_7$ form an SU(2) subgroup; that is, show that $[2T_2, 2T_5] = i2T_7$, etc. To save time, only do two of the three commutators. How does the 3 representation of SU(3) break into representations under this subgroup?

2. Of the eight generators, two of them can be diagonalized simultaneously (normally chosen as T_3 and T_8). In this problem, you will organize the others into pairs, comparable to the "raising" and "lowering" operators for SU(2)
   (a) Combine the remaining six generators, such that the commutation relations of the resulting combinations with T_3 and T_8 always come out proportional to the resulting generators. Here is one of them done for you:

$$T_A = T_1 + iT_2, \quad \text{then} \quad [T_3, T_A] = +1T_A \quad \text{and} \quad [T_8, T_A] = 0T_A$$

   (b) For each of the six generators you just worked out, plot on a 2D graph the resulting coefficients when you commute with T_3 and T_8. The first one is done for you. This diagram is called a *root diagram*.
   (comment: technically, a root diagram would also include two zero roots, corresponding to the two generators T_3 and T_8 themselves, which commute with each other)