Physics 745 - Group Theory

Homework Set 27

Due Wednesday, April 15

1. This problem has to do with demonstrating that according the isospin symmetry, the three $|\Sigma\rangle$'s all have the same mass.
   a) Work out the effects of the isospin operators $\mathcal{I}_\pm$ on all three of the $|\Sigma\rangle$ states.
   b) Assuming that isospin commutes with the mass portion of the Hamiltonian, show that all four of the $|\Sigma\rangle$'s have the same mass.

2. There is an excited set of two baryons $|N^+\rangle$ and $|N^0\rangle$ that have the same isospin properties of the neutron and proton. They can decay, in principle, into a $|\Delta\pi\rangle$ combination of two particles.
   a) Suppose that the Hamiltonian that performs this transition takes the form
   $$H|N^+\rangle = a|\Delta^{++};\pi^-\rangle + b|\Delta^+;\pi^0\rangle + c|\Delta^0;\pi^+\rangle$$
   Find the relative sizes of the factors *a*, *b*, and *c*. I recommend doing this by letting $\mathcal{I}_+$ act on both sides.
   b) Calculate the relative rate for the decay rates $\Gamma(N^+ \to \Delta^{++}\pi^-)$, $\Gamma(N^+ \to \Delta^+\pi^0)$, and $\Gamma(N^+ \to \Delta^0\pi^+)$.
   c) Now I want you to figure out how the N^0 decays. Using isospin symmetry, determine that the same interaction discussed in part (a) also leads to three decay processes for the N^0. Find the relative probability amplitudes for these processes, and predict the corresponding decay rates, and show how they relate to those found in part (b).