Physics 780 – General Relativity

Homework A

1. Using the convention that $c = 1$, rewrite the following quantities (on personal ones, you can lie, but your lie has to be plausible)
   (a) Your age in m
   (b) The length of your foot in ns
   (c) The Schwarzschild radius of the Sun, given by $2GM$, in km

2. A particle moves in a helical path given by $(x, y, z) = (R\cos(\omega t), R\sin(\omega t), vt)$. Find a relation between the coordinate time t and the proper time τ, and then rewrite all four coordinates in terms of τ.

3. A general Lorentz transformation is a 4×4 matrix satisfying $\eta_{\mu\nu} = \eta_{\alpha\beta}\Lambda^{\alpha}{}_{\mu}\Lambda^{\beta}{}_{\nu}$, which can be written in matrix form as $\eta = \Lambda^T \eta \Lambda$
   (a) Using the formulas $\det(AB) = \det(A)\det(B)$ and $\det(A^T) = \det(A)$, show that $\det(\Lambda) = \pm 1$.
   (b) Using this equation in the case $\mu = \nu = 0$, show that $\Lambda^0{}_0 \geq 1$ or $\Lambda^0{}_0 \leq -1$.
   (c) Argue that if you start with the identity Lorentz transformation ($\Lambda = \mathbf{1}$), and then continuously change it, by making small rotations or boosts, the sign of $\det(\Lambda)$ and $\Lambda^0{}_0$ will never change. Call these Lorentz transformations *proper* Lorentz transformation.
   (d) Show that time reversal $\Lambda = \mathcal{T} = \text{diag}(-1,1,1,1)$ parity, $\Lambda = \mathcal{P} = \text{diag}(1,-1,-1,-1)$, and the combination $\Lambda = \mathcal{PT}$ are improper Lorentz transformations.

4. The concept of future and past do not work exactly the same in special relativity, but some things are the same. We will say that a point x^μ is in the absolute future of another point y^μ if $x^0 - y^0 > |\mathbf{x} - \mathbf{y}|$. In standard physics, the future of the future is the future. Show that in relativity, if x is in the future of y and y is in the future of z, then x is in the future of z. You may want to look up the triangle inequality.