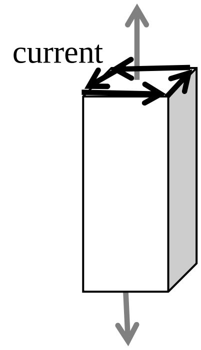
Physics 712
Chapter 6 Solutions

4. **[5] Consider a cylinder of arbitrary cross-sectional shape, such as a square, circle, or other similar shape. This cylinder will be infinitely long in the *z*-direction. It will have a surface current *K*, with units A/m, running around it in a counter-clockwise direction as viewed from above.**



(a) In which direction(s) can you translate this cylinder and leave it unchanged? What can you conclude about the resulting magnetic field?

The cylinder can be translated along the *z*-axis without changing the problem. Therefore, all components of the magnetic field must be independent of *z*, and if we are working in Cartesian coordinates, we could write

$$\mathbf{B}(\mathbf{r}) = B_x(x,y)\hat{\mathbf{x}} + B_y(x,y)\hat{\mathbf{y}} + B_z(x,y)\hat{\mathbf{z}}$$

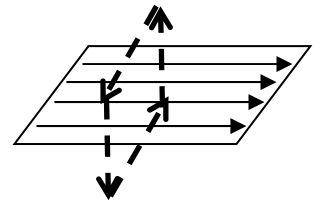
(b) Across which plane can you reflect this current and leave it unchanged? Based on this, which components of the magnetic field must vanish?

You can reflect it across the *xy*-plane, which reverses *z* and leaves *x* and *y* unchanged. Keeping in mind that **B** is a pseudovector, we would have, under these circumstances,

$$\mathbf{B}(\mathbf{r}) \to -B_x(x,y)\hat{\mathbf{x}} - B_y(x,y)\hat{\mathbf{y}} + B_z(x,y)\hat{\mathbf{z}}$$

Since the magnetic field must be unchanged, we conclude $B_x = B_y = 0$, so there is only magnetic field in the *z*-direction.

5. **[10] Consider an infinite plane of surface current in the plane $z = 0$ flowing in the direction $\mathbf{K} = K\hat{\mathbf{x}}$, where K has units of A/m.**



(a) **Which direction(s) can you translate this current and leave it unchanged? What conclusions can you draw about the B-field?**

The current can be translated in the *x* or *y*-direction and leave the problem unchanged, so we conclude that **B** can only be a function of *z*, so $\mathbf{B} = B_x(z)\hat{\mathbf{x}} + B_y(z)\hat{\mathbf{y}} + B_z(z)\hat{\mathbf{z}}$.

(b) **By reflecting this problem across the $y = 0$ plane, which of the components of B can you conclude must vanish?**

When you reflect across the $y = 0$ plane, you reverse *y* and leave *x* and *z* unchanged. But since **B** is a pseudovector, this would change **B** to $\mathbf{B} \to \mathbf{B} = -B_x(z)\hat{\mathbf{x}} + B_y(z)\hat{\mathbf{y}} - B_z(z)\hat{\mathbf{z}}$. Since the magnetic field is unchanged, the *x* and *z* components must vanish, so $\mathbf{B} = B_y(z)\hat{\mathbf{y}}$.

(c) **By reflecting this problem across the $z = 0$ plane, show that you can relate the field above the plane to the field below the plane.**

When you reflect across $z = 0$, *z* changes sign, and since **B** is psudovector, so does B_y and B_x (but not B_z). This tells us $\mathbf{B} \to \mathbf{B} = -B_y(-z)\hat{\mathbf{y}}$, which tells us $B_y(-z) = -B_y(z)$.

(d) **Using an appropriate Ampere loop, find B everywhere.**

Consider the loop sketched above, which is at height *h* above and below the plane on the top and bottom, and has length *L* in the *y*-direction. The current flowing through this loop will be *KL*, so by Ampere's law for loops, we have

$$\mu_0 KL = \oint \mathbf{B} \cdot d\mathbf{l} = -B_y(h)L + 0 + B_y(-h)L + 0 = -2LB(h).$$

Solving for $B(h)$, we find for $h > 0$ that $B_y(h) = -\tfrac{1}{2}\mu_0 K$, from which we conclude

$$\mathbf{B} = -\tfrac{1}{2}\mu_0 K \operatorname{sgn}(z)\hat{\mathbf{y}},$$

Where $\operatorname{sgn}(z) = +1$ if $z > 0$ and $\operatorname{sgn}(z) = -1$ if $z < 0$.