Physics 712
Solutions to Chapter 11 Problems

1. [10] We want to consider the effect of two boosts along the *x*-axis. The following hyperbolic identities will prove useful:

$$\cosh(\phi_1 \pm \phi_2) = \cosh\phi_1 \cosh\phi_2 \pm \sinh\phi_1 \sinh\phi_2 , \qquad \text{and} \qquad \tanh(\phi_1 \pm \phi_2) = \frac{\tanh\phi_1 \pm \tanh\phi_2}{1 \pm \tanh\phi_1 \tanh\phi_2} .$$
$$\sinh(\phi_1 \pm \phi_2) = \sinh\phi_1 \cosh\phi_2 \pm \cosh\phi_1 \sinh\phi_2 ,$$

(a) For two successive boosts with rapidity ϕ_1 and ϕ_2 find the equivalent rapidity ϕ_{tot}.

We simply combine two boosts using these formulas, which gives us

$$\Lambda_{tot} = \Lambda_2\Lambda_1 = \begin{pmatrix} \cosh\phi_1 & -\sinh\phi_1 & 0 & 0 \\ -\sinh\phi_1 & \cosh\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cosh\phi_2 & -\sinh\phi_2 & 0 & 0 \\ -\sinh\phi_2 & \cosh\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh\phi_1 \cosh\phi_2 + \sinh\phi_1 \sinh\phi_2 & -\cosh\phi_1 \sinh\phi_2 - \sinh\phi_1 \cosh\phi_2 & 0 & 0 \\ -\cosh\phi_1 \sinh\phi_2 - \sinh\phi_1 \cosh\phi_2 & \cosh\phi_1 \cosh\phi_2 + \sinh\phi_1 \sinh\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\phi_1+\phi_2) & -\sinh(\phi_1+\phi_2) & 0 & 0 \\ -\sinh(\phi_1+\phi_2) & \cosh(\phi_1+\phi_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

Obviously, $\phi_{tot} = \phi_1 + \phi_2$.

(b) For two successive boosts with velocity v_1 and v_2 find the equivalent velocity v_{tot}.

We know that $v = c\tanh\phi$, so we have

$$v_{tot} = c\tanh\phi_{tot} = c\tanh(\phi_1+\phi_2) = c\frac{\tanh\phi_1 + \tanh\phi_2}{1+\tanh\phi_1 \tanh\phi_2} = c\frac{v_1/c + v_2/c}{1+v_1v_2/c^2} = \frac{v_1+v_2}{1+v_1v_2/c^2} .$$