Physics 712

Chapter 0 Solution

1. **Consider the vector function $\mathbf{x}/|\mathbf{x}|^3$. By direction computation show that it has no curl for $\mathbf{x} \neq 0$. By integrating it over a small spherical volume, show that it also has no curl for $\mathbf{x} = 0$.**

As in class, we first write the function in spherical coordinates, so that $\mathbf{x}/|\mathbf{x}|^3 = \hat{\mathbf{r}} r^{-2}$. We then simply calculate the curl:

$$\nabla \times \left(\frac{1}{r^2} \hat{\mathbf{r}} \right) = \hat{\boldsymbol{\theta}} \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} \left(\frac{1}{r^2} \right) - \hat{\boldsymbol{\varphi}} \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{1}{r^2} \right) = 0.$$

So far so good! But we have to be careful at $r = 0$, since the function is large there. We integrate the curl over a small region to give

$$\int_V \nabla \times \left(\frac{1}{r^2} \hat{\mathbf{r}} \right) d^3\mathbf{x} = \int_S \hat{\mathbf{n}} \times \left(\frac{1}{r^2} \hat{\mathbf{r}} \right) da = \frac{1}{r^2} \int_S \hat{\mathbf{r}} \times \hat{\mathbf{r}} da = 0.$$

Since the integral is zero, there is no need to add a delta function at zero. So $\nabla \times \frac{\mathbf{x}}{|\mathbf{x}|^3} = 0$.