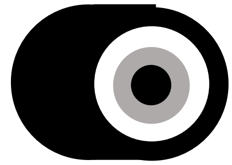
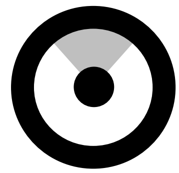
Physics 712
Chapter 4 Problems

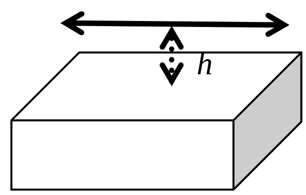
1. A long cylindrical cavity with inner radius a and outer radius c is filled with a dielectric with dielectric constant ε from radius a to radius b and with vacuum from radius b to radius c. If there is charge per unit length λ on the inner conductor, find the electric field everywhere between the cylinders, the potential difference between the inner and outer conductors, and the bound charge density on the surface, if any, of the dielectric at radius b.



2. A spherical cavity of outer radius a and inner radius b is partially filled with a dielectric with dielectric constant ε in the region $0 < \theta < \frac{1}{3}\pi$. A charge Q is on the inner sphere. Find the electric field everywhere between the spheres, the voltage difference between the inner and outer spheres, and the bound charge density on the surface, if any, of the dielectric at $\theta = \frac{1}{3}\pi$.



3. An infinite line charge with charge per unit length $\lambda = 0$ parallel to the x-axis a distance h above a semi-infinite dielectric with dielectric constant ε. Find the force per unit length on the line charge. Find the bound surface charge density σ_b on the surface of the dielectric.



4. A dielectric sphere with dielectric constant ε of radius a lies at the origin in a background potential (in the absence of the sphere) of the form $\Phi(\mathbf{x}) = \lambda xy$.
   (a) Write the background potential in terms of spherical harmonics times powers of r.
   (b) Write a reasonable conjecture for the form of the potential in the regions $r < a$ and $r > a$. Your conjecture should automatically satisfy $\nabla^2\Phi = 0$ within each of these regions. It may contain unknown constants.
   (c) By matching suitable boundary conditions, determine the value of any unknown constants.

5. For problem 4.1, find the total energy if the cylinder has length L. For problem 4.2, find the total energy. In each case, show that the answer is equivalent to $W = \frac{1}{2}Q\Delta\Phi$.