Physics 712
Chapter 11 Problems

1. We want to consider the effect of two boosts along the *x*-axis. The following hyperbolic identities will prove useful:

$$\cosh(\phi_1 \pm \phi_2) = \cosh\phi_1 \cosh\phi_2 \pm \sinh\phi_1 \sinh\phi_2, \quad \sinh(\phi_1 \pm \phi_2) = \sinh\phi_1 \cosh\phi_2 \pm \cosh\phi_1 \sinh\phi_2, \quad \text{and} \quad \tanh(\phi_1 \pm \phi_2) = \frac{\tanh\phi_1 \pm \tanh\phi_2}{1 \pm \tanh\phi_1 \tanh\phi_2}.$$

   (a) For two successive boosts with rapidity ϕ_1 and ϕ_2 find the equivalent rapidity ϕ_{tot}.

   (b) For two successive boosts with velocity v_1 and v_2 find the equivalent velocity v_{tot}.

2. Consider a particle moving along the *x*-axis whose 4-velocity is given at proper time τ by $U^\mu = c(\cosh\phi, \sinh\phi, 0, 0)$, where ϕ is an unknown function of time.

   (a) Check that $U \cdot U = c^2$. Find the proper acceleration $a(\tau)$ at time τ for an arbitrary function $\phi(\tau)$.

   (b) Suppose $a(\tau) = g$, a constant. Assuming the particle starts at the origin at $\tau = 0$ and is initially at rest, find $\phi(\tau)$, $U(\tau)$ and $x(\tau)$.

   (c) How much proper time (in years) would it take to get to Alpha Centauri (4.3 *c*·y), the center of our galaxy (2.6×10^4 *c*·y), or the edge of the visible universe (4.5×10^{10} *c*·y) if you start at rest and accelerate in a straight line at proper acceleration g = 9.8 m/s^2?

3. A pion (mass m_π) at rest decays to a muon (mass m_μ) and a neutrino (mass 0). Find the energies of the two final particles.

4. A particle of mass *m* and charge *q* is in the presence of constant electric and magnetic fields $\mathbf{E} = E\hat{\mathbf{x}}$ and $\mathbf{B} = B\hat{\mathbf{z}}$.

   (a) Write out explicitly all four components of the equation for $\dot{U}^\mu$, where dot stands for $d/d\tau$. Find an equation for $\ddot{U}^1$.

   (b) What is the general solution for $U^1(\tau)$ if $E < cB$? Argue that it will exhibit periodic behavior (in τ), and find the period.

   (c) Repeat part (b) if $E > cB$. Will it be periodic in this case?

5. Consider a line of charge with linear charge density λ arranged, in a primed frame, along the *y'*-axis at rest. Write the electric field at all points in Cartesian coordinates in the primed frame. Now, consider a line of charge with the same linear charge density, parallel to the *y*-axis, but this time moving in the +*x* direction at velocity *v*. Find the electric and magnetic fields everywhere in the unprimed frame.