Physics 712 – Electricity and Magnetism

Equations for Midterm Exam

The following equations you should have memorized

Cartesian coordinates:
Gradient, Divergence, Curl and Laplacian
3D Integrals:
In Cartesian and spherical

Electric Field from Charges

$$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int \rho(\mathbf{x}')\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}d^3\mathbf{x}',$$

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}'$$

Statics Equations

$\nabla\cdot\mathbf{D} = \rho, \quad \nabla\times\mathbf{E} = 0$

$\mathbf{D}_\perp, \mathbf{E}_\parallel$ continuous

$\nabla\cdot\mathbf{B} = 0, \quad \nabla\times\mathbf{H} = \mathbf{J}$

$\mathbf{B}_\perp, \mathbf{H}_\parallel$ continuous

Divergence Theorem

$$\int_V \nabla\cdot\mathbf{A}\, d^3\mathbf{x} = \int_S \mathbf{A}\cdot\hat{\mathbf{n}}\, da$$

Linear media

$\mathbf{D} = \varepsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}$

$\varepsilon = \varepsilon_0, \mu = \mu_0$ in vacuum

Electric Potential

$\mathbf{E} = -\nabla\Phi$

$\nabla^2\Phi = -\rho/\varepsilon$

Gauss's Law

$$\int_S \mathbf{D}\cdot\hat{\mathbf{n}}\, da = q(V)$$

$$\int_S \mathbf{B}\cdot\hat{\mathbf{n}}\, da = 0$$

Lorentz Force

$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$

Faraday's Law

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

Method of images (plane):

$q' = -q, \quad h' = -h$

Conductors:

$\mathbf{E} = 0$

$\Phi =$ constant

Ampere's Law (statics)

$$\oint \mathbf{H}\cdot d\mathbf{l} = I$$

Vector Potential:

$\mathbf{B} = \nabla\times\mathbf{A}$

Energy Density

$w = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$

Charge Conservation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{J} = 0$$

The following equation you need not memorize, but you should be able to use them if given to you:

Green's Function

$$\nabla'^2 G(\mathbf{x},\mathbf{x}') = -4\pi\delta^3(\mathbf{x}-\mathbf{x}'), \quad G(\mathbf{x},\mathbf{x}')\big|_{\mathbf{x}'\in S_D} = 0, \quad \hat{\mathbf{n}}'\cdot\nabla' G(\mathbf{x},\mathbf{x}')\big|_{\mathbf{x}'\in S_N} = 0$$

$$4\pi\Phi(\mathbf{x}) = \frac{1}{\varepsilon_0}\int_V G(\mathbf{x},\mathbf{x}')\rho(\mathbf{x}')d^3\mathbf{x}' + \int_{S_N} G(\mathbf{x},\mathbf{x}')\frac{\partial}{\partial n'}\Phi(\mathbf{x}')da' - \int_{S_D}\Phi(\mathbf{x}')\frac{\partial}{\partial n'}G(\mathbf{x},\mathbf{x}')da'$$

Spherical and Cylindrical coordinates:
Gradient, Divergence, Curl and Laplacian, and 3D integrals

Polarization and Magnetization

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

$$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$$

$$\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}}$$

Conducting Surface

$$\mathbf{D} = \sigma\hat{\mathbf{n}}$$

Method of Images for Spheres

$$q' = -\frac{aq}{r}, \quad r' = \frac{a^2}{r}$$

Complete functions for 1D

Periodic: $e^{2\pi inx/a}$

Boundary: $\sin(\pi nx/a)$

Bessel's Equation

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} - \frac{m^2}{\rho^2}R + k^2R = 0$$

$$R = J_m(kr) \quad \text{or} \quad R = N_m(kr)$$

Spherical Harmonics

Multipole Expansion

$$q_{lm} = \int Y_{lm}^*(\theta,\phi)r^l\rho(\mathbf{x})d^3\mathbf{x}$$

$$\Phi(\mathbf{x}) = \frac{1}{\varepsilon_0}\sum_{l=0}^{\infty}\frac{1}{(2l+1)r^{l+1}}\sum_{m=-l}^{l}q_{lm}Y_{lm}(\theta,\phi)$$

Normalization

$$\int_0^a \sin(\pi nx/a)\sin(\pi mx/a)dx = \tfrac{1}{2}a\delta_{nm}$$

$$\int_0^a J_m(x_{mn}\rho/a)J_m(x_{mn'}\rho/a)dx = \tfrac{1}{2}a^2J_{m+1}^2(x_{mn})\delta_{nn'}$$

General Solution of Laplace in Spherical

$$\Phi(\mathbf{x}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left(A_{lm}r^l + B_{lm}r^{-l-1}\right)Y_{lm}(\theta,\phi)$$

Method of Images for Dielectrics

outside: $q' = \dfrac{\varepsilon_0 - \varepsilon}{\varepsilon + \varepsilon_0}q$ at $-h$

inside: $q'' = \dfrac{2\varepsilon}{\varepsilon + \varepsilon_0}q$ at $+h$

Biot-Savart Law

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I}{4\pi}\int\frac{d\mathbf{l}'\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int\mathbf{J}(\mathbf{x}')\times\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}d^3\mathbf{x}'$$

Force on a Wire

$$\mathbf{F} = \int\mathbf{J}\times\mathbf{B}d^3\mathbf{x}$$

Magnetic Dipoles

$$\mathbf{m} = IA\hat{\mathbf{n}}$$

$$\mathbf{m} = \tfrac{1}{2}\int\mathbf{x}'\times\mathbf{J}(\mathbf{x}')d^3\mathbf{x}'$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{3(\hat{\mathbf{r}}\cdot\mathbf{m})\hat{\mathbf{r}} - \mathbf{m}}{r^3}$$

Energy

$$W = \tfrac{1}{2}\int\rho\Phi\, d^3\mathbf{x}$$

$$W = \tfrac{1}{2}\int\mathbf{J}\cdot\mathbf{A}\, d^3\mathbf{x}$$

B-field from Straight Wire

$$\mathbf{B} = \frac{\mu_0 I}{4\pi\rho}\hat{\boldsymbol{\varphi}}\left[\cos\theta_b + \cos\theta_a\right]$$