Physics 310/610 – Cosmology

Solution Set R

1. [10] For each of the decays below, the decay is a strong decay.

(a) Consider the collision $p^+ + K^- \to K^+ + K^0 + \Omega$. Based on what you know, what is the baryon number, charge, and strangeness of the Ω? Give an argument that the Ω is none of the particles listed.

On the left we have one baryon, but no baryon on the right, so it must be a baryon. The total charge on the left is zero, which must match the total charge on the right, so to balance the K^+, the Ω must be charge –1. Finally, on the left, the proton has no strangeness and the K^- has strangeness $S = -1$, while each of the kaons on the right have strangeness $S = +1$. If we let x be the strangeness of the Ω, then we have $0-1=+1+1+x$, so $x=-3$. There is no particle, let alone a baryon, with strangeness –3, so this is a particle not in our table.

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(b) For the decay $\Lambda^{*0} \to \pi^- + X$, what is the baryon number, charge, and strangeness of the *X*? Find an upper limit on the mass of the *X*. Based on this, determine which particle *X* must be.

We have a baryon on the left, but the pion isn't a baryon, so *X* must be a baryon. To balance charge, it must have charge +1. Since the Λ^{*0} has strangeness –1 and the pion has not strangeness, the *X* must also be strangeness –1. Finally because we have a decay, we must have $m_{\Lambda^*} > m_\pi + m_X$, so $m_X < m_{\Lambda^*} - m_\pi = (1406-139)\ \text{MeV}/c^2 = 1267\ \text{MeV}/c^2$. The only particle that fits this description is the Σ^+, so that must be what it is.

(c) The Δ^{++} always decays to two particles, and it is always the same two particles. Figure out which two particles it is, and give an argument.

To conserve baryon number it must decay to a baryon and a meson. All the mesons weigh at least 135 MeV/c^2, so to conserve energy, this means the baryon must be lighter than $(1231-135)\ \text{MeV}/c^2 = 1096\ \text{MeV}/c^2$, which narrows it down to the proton and neutron. Similarly, since the lightest baryon is 938 MeV/c^2, the meson must be no heavier than $(1231-938)\ \text{MeV}/c^2 = 293\ \text{MeV}/c^2$, which narrows it down to the three pions. The only decay that conserves charge would then be $\Delta^{++} \to p^+ + \pi^+$.

2. **[10] For each of the processes below, categorize the process as strong, electromagnetic, weak, or impossible. For this problem, you do not need to show your work.**

(a) $p^+ \to e^+ + \gamma$ **impossible**

(b) $n^0 \to p^+ + e^-$ **impossible**

(c) $K^0 \to \pi^+ + \pi^-$ **weak**

(d) $\Sigma^+ \to n^0 + K^+$ **impossible**

(e) $\pi^+ \to \mu^+ + \nu_1$ **weak**

(f) $p^+ + e^- \to n^0 + e^+$ **impossible**

(g) $\Sigma^0 \to \Lambda^0 + \gamma$ **electromagnetic**

(h) $\Delta^+ \to p^+ + \pi^0$ **strong**

The proton decay (a) is impossible because there is a baryon on the left and no baryon on the right. The neutron decay (b) is impossible because there are a total of three fermions on the two sides.

The kaon decay (c) has no baryons, no fermions, and charge is conserved. The mass of the kaon exceeds the two pions, so it is possible. But since strangeness isn't conserved, it is weak.

The charged sigma decay (d) has more mass on the right than the left, and since it is a decay, it is impossible.

The pion decay (e) has no baryons, two fermions on the right (and none on the left) and conserves charge, so it is possible. Since it involves a neutrino, it is weak. The proton plus electron (f) is impossible because it violates conservation of charge. If you changed the positron to a neutrino it would become possible.

The neutral sigma decay (g) conserves baryon number charge, and the lambda is lighter than the sigma (and the photon is massless) and it has one fermion on each side, so this is fine. Since one of the particles (the photon) is not strongly interacting, it's not strong, and since it doesn't have any neutrinos, it must be electromagnetic.

For the delta baryon decay, you have one baryon on each side, one fermion on each side, and the delta has more mass than the proton plus pion. So it is possible. Since all the particles are strong, it is a strong interaction.

Graduate Problem: Do this problem only if you are in PHY 610.

3. **[15] In this problem we will discuss the difference between collider physics and cosmic ray physics.**

(a) Suppose a collider collides two particles with equal mass *m*, head on, with equal energy *E*. Assuming they combine into a single particle, what would be the particle mass *M* of the resulting particle? (This part is trivial)

We will use the formula $E^2 = p^2c^2 + m^2c^4$. For two particles colliding back to back, they will each have energy E and momentum p, but the momenta will have opposite signs, so the total energy is just $2E$ and the total momentum zero. Bottom line: we have $(2E)^2 = 0^2 + M^2c^4$, so $Mc^2 = 2E$.

(b) Now suppose instead we collide two particles with mass *m*, but one is at rest, and the other has energy *E*. What would be the mass *M* of the resulting particle? (This part is *not* trivial)

One of the particles has energy mc^2 and momentum zero. The other has momentum p and energy E. We therefore can get the invariant mass using

$$M^2c^4 = E_{tot}^2 - p_{tot}^2c^2 = \left(E + mc^2\right)^2 - p^2c^2 = E^2 + 2mc^2E + m^2c^4 - p^2c^2 .$$

We now note that the moving particle satisfies $E^2 = p^2c^2 + m^2c^4$, so we can use this to simplify the expression to

$$M^2c^4 = m^2c^4 + 2mc^2E + m^2c^4 = 2mc^2\left(E + mc^2\right).$$

(c) The Large Hadron Collider (LHC) collides pairs of protons head on with energy 7 TeV each. Suppose instead, a cosmic proton of energy *E* collides with a stationary proton. How large would *E* have to be to achieve the same invariant mass *M* for the collision?

The LHC collides with an invariant mass of $Mc^2 = 2E = 14{,}000 \text{ GeV}$. To reach this energy in a cosmic ray proton would have to have

$$2mc^2\left(E + mc^2\right) = \left(14{,}000 \text{ GeV}\right)^2 = 1.96\times10^8 \text{ GeV}^2$$

We now substitute the mass of the proton, 0.938 GeV/c^2, to find

$$2\left(0.938 \text{ GeV}\right)\left(E + 0.938 \text{ GeV}\right) = 1.96\times10^8 \text{ GeV}^2 ,$$

$$E + 0.938 \text{ GeV} = \frac{1.96\times10^8 \text{ GeV}^2}{2\left(0.938 \text{ GeV}\right)} = 1.045\times10^8 \text{ GeV},$$

$$E = 1.045\times10^8 \text{ GeV} = 1.045\times10^{17} \text{ eV}.$$

Despite this seemingly absurdly high number, there *are* cosmic rays at this energy, and beyond. They are just very rare. According to Wikipedia, there are events at least as high as $E = 3\times10^{20} \text{ eV}$, a few thousand times more energetic than this, though those may have been iron nuclei, not protons.

Incidentally, if you enjoy metric prefixes, the energy listed here is about 0.1 EeV, and the highest observed are around 0.3 ZeV.