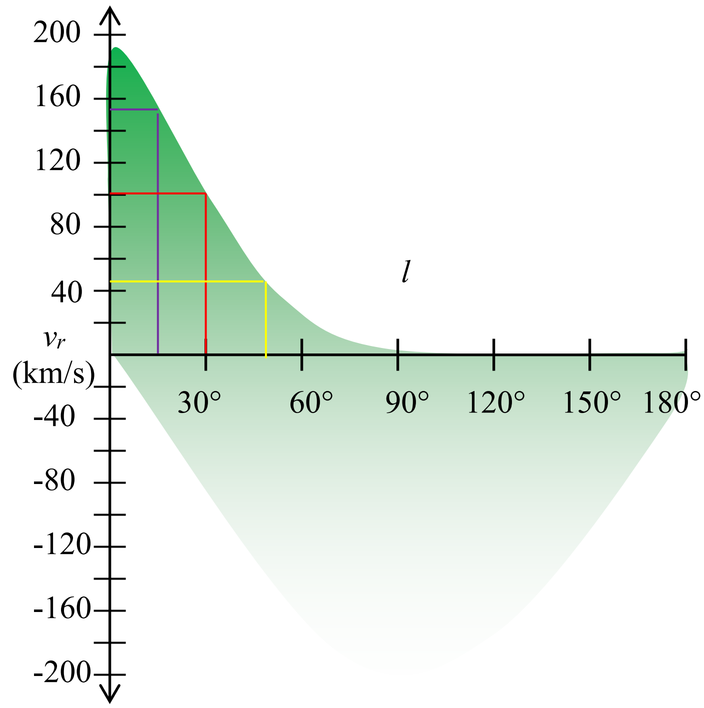
Physics 310/610 – Cosmology

Solution Set I

1. **[15] An alien race is circling their galaxy in a circular orbit at *r* = 8 kpc at a velocity of *V* = 200 km/s. They measure the 21 cm line in the plane of their galaxy as a function of galactic longitude *l*. Because they are seeing through many different clouds of gas in the galactic plane, they will see different redshifts, and therefore they get a band of values, as sketched at right. Because they are lazy, they only do half of the plane.**



   (a) [3] If they want to use the tangent method to find the circular orbital velocity *V*, at what angle *l* should they examine the maximum recessional velocity v_r to determine the velocity of gas at *r* = 2 kpc, 4 kpc, and 6 kpc from the center?

The angle you want to look at is the angle where the line of sight is tangent to a circle of radius *r*. As demonstrated in class, this is given by $r = r_0 \sin l$, so $\sin l = r/r_0$ or $l = \sin^{-1}(r/r_0)$. Substituting in the specific numbers, we find

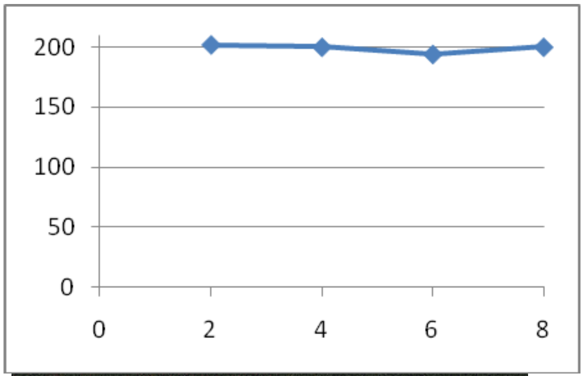
$$l_2 = \sin^{-1}\tfrac{1}{4} = 14.5°,$$
$$l_4 = \sin^{-1}\tfrac{1}{2} = 30.0°,$$
$$l_6 = \sin^{-1}\tfrac{3}{4} = 48.6°.$$

   (b) [9] Estimate the rotational velocity at each of these angles

We use the formula from the notes, namely, $V(r_0 \sin l) = v_r + V_0 \sin l$. From the graph, we estimate the radial velocities at 152, 100, and 44 km/s respectively, so we have

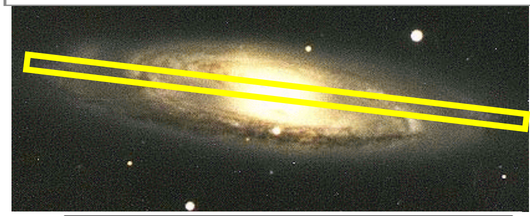
$$V(2\text{ kpc}) = (152\text{ km/s}) + 200 \cdot \sin(14.5°) = 202\text{ km/s},$$
$$V(4\text{ kpc}) = (100\text{ km/s}) + 200 \cdot \sin(30.0°) = 200\text{ km/s},$$
$$V(6\text{ kpc}) = (44\text{ km/s}) + 200 \cdot \sin(48.6°) = 194\text{ km/s}.$$

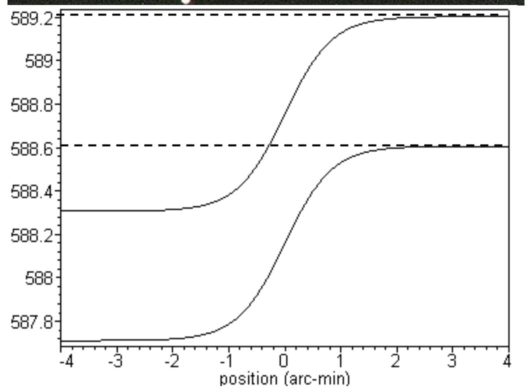
(c) [3] Plot/sketch these rotational velocities as a function of *r*, throwing in *V* = 200 km/s at *r* = 8 kpc. Does this galaxy show evidence for dark matter?



A sketch appears at right. That curve looks pretty flat, indicating the presence of dark matter.

2. [15] The galaxy at right is put through a slit spectrometer, which measures the spectrum at each point along the length of the light box shown at right. It is found that there are some spectral lines around 588 nm. This is presumably the result of the sodium-D lines, which occur in the laboratory at wavelengths of 588.9950 nm and 589.5924 nm.





(a) [1] Which of the observed lines is which?

The higher one must correspond to the longer wavelength. That was easy.

(b) [8] At each extreme of the galaxy (left end of graph, right end of graph), what is the velocity of that side of the galaxy, and is it towards us or away from us?

To determine this, we need to use the non-relativistic formula

$$\lambda/\lambda_0 = 1 + z = 1 + v_r/c$$

This allows us to determine the recessional velocity on each side of the galaxy. We can use either wavelength on each end. We have

$$\text{Left:}\quad v_r = c\left(\frac{\lambda}{\lambda_0} - 1\right) = \left(2.998\times10^8 \text{ m/s}\right)\left(\frac{588.31}{589.59} - 1\right) = -6.52\times10^5 \text{ m/s},$$

$$\text{Left:}\quad v_r = c\left(\frac{\lambda}{\lambda_0} - 1\right) = \left(2.998\times10^8 \text{ m/s}\right)\left(\frac{587.71}{589.00} - 1\right) = -6.54\times10^5 \text{ m/s},$$

$$\text{Right:}\quad v_r = c\left(\frac{\lambda}{\lambda_0} - 1\right) = \left(2.98\times10^8 \text{ m/s}\right)\left(\frac{589.21}{589.59} - 1\right) = -1.93\times10^5 \text{ m/s},$$

$$\text{Right:}\quad v_r = c\left(\frac{\lambda}{\lambda_0} - 1\right) = \left(2.98\times10^8 \text{ m/s}\right)\left(\frac{588.61}{589.00} - 1\right) = -1.96\times10^5 \text{ m/s}.$$

Since all numbers came out negative, it is clear both sides of the galaxy are moving towards us (negative recessional velocity): the left side at about 653 km/s, the right side at about 195 km/s.

(c) [4] Average the two velocities to estimate the velocity of the galaxy towards or away from us. What is the orbital velocity (velocity relative to the center of the galaxy) of the stars at the two extreme edges of the galaxy?

The average velocity is about 424 km/s, towards us. Now, a naïve estimate of the galaxy's rotational velocity would be the *difference* between this speed and the speeds at the ends, which is 229 km/s. This would be true if it were *exactly* edge on to us.

However, it is a little bit tilted. If we assume the disk is a circle, well, then the disk looks like it is about 4.5 cm on the long axis and 1.1 cm on the short axis. This implies that it's tilted compared to the line of sight by about $\sin^{-1}(1.1/4.5) = 14°$. Then the velocity towards us or away from us will be $V\cos(14°)$, so the actual rotational velocity is

$$V = \frac{229 \text{ km/s}}{\cos(14°)} = 236 \text{ km/s}$$

The correction really is hardly worth it, since I don't believe the numbers.

Incidentally, this is actually a picture of M31, the Andromeda Galaxy, and it actually moves towards us at 301 km/s (as viewed from the Sun). But I made up the numbers, so no surprise it came out wrong. I bet I got the scale wrong too, but so what?

(d) [2] Is there evidence for dark matter in this galaxy?

The hallmark of dark matter in spiral galaxies is flat rotation curves. These curves flatten out at large radii, so yes, it really does contain dark matter.

PHYSICS 610: There are no graduate problems for this homework