Physics 310/610 – Cosmology

Homework Set T

1. One thing we haven't discussed, up to now, is pressure. All particles in the universe contribute to the pressure, but neutrinos are irrelevant because they don't interact with anything. We will look first at recombination, around $z = 1090$, $k_B T = 0.256$ eV.
   (a) The current density of baryons and dark matter are about $\rho_{b0} = 4.196\times10^{-28}\ \text{kg/m}^3$ and $\rho_{d0} = 2.235\times10^{-27}\ \text{kg/m}^3$. What was their density at recombination? What was the number density, assuming the baryons are hydrogen atoms, and dark matter has a mass of $m_d = 10^3\ \text{GeV}/c^2$?
   (b) Assuming cold dark matter is the same temperature as baryons, they should each contribute a pressure of approximately $P = k_B nT$. In fact, the cold dark matter particles are almost certainly colder. Which contribution is more important? Calculate $P = k_B nT$ for the more important component.
   (c) In addition, the radiation itself generates some pressure, given by $P = \frac{1}{3}u$, where u is the energy density. Calculate the radiation pressure. Of the three components, which is most important?
   (d) By what factor does the pressure felt by the baryons drop from before recombination (photons + baryons) compared to after (baryons only)?

2. At the time of recombination, $k_B T = 0.256\ \text{eV}$, the atoms will all be in motion.
   (a) According to thermodynamics, a typical thermal velocity for a non-relativistic particle is given by $E_{kin} = \frac{3}{2}k_B T$. Estimate the typical velocity of a hydrogen atom at this time.
   (b) Multiply this speed by the age of the universe at this time to get an approximate distance d that an atom would move at the time of recombination.
   (c) A sphere of radius d found in part (b) will tend to have its density fluctuations wiped out (or at least diminished) by the atoms wandering off. Use the density of the dark matter ρ_d from problem 1b and the distance you just found to get the mass contained within this sphere. This should be the smallest structures that form, and the first structures that form. Compare to the size of a globular clusters, $10^4 - 10^7\ M_\odot$.

3. At the time of matter radiation equality $z = 3400$, any fluctuations that are larger than the "horizon size" at the time will have a big disadvantage in forming structures.
   (a) The horizon size is simply $d = ct$. Find the horizon size at this time in kpc.
   (b) Scale it up to the present. Give the answer in Mpc. Compare to the size of the Laniakea Supercluster, 160 Mpc.

Graduate Problem: There are no graduate problems for this homework.