Physics 310 – Cosmology

Homework Set N

1. The density of ordinary matter (baryons), dark matter, radiation, and cosmological constant are currently about

$$\rho_{b0} = 4.196\times10^{-28}\ \text{kg/m}^3, \quad \rho_{r0} = 4.642\times10^{-31}\ \text{kg/m}^3,$$
$$\rho_{d0} = 2.235\times10^{-27}\ \text{kg/m}^3, \quad \rho_{\Lambda 0} = 5.966\times10^{-27}\ \text{kg/m}^3.$$

The baryons and dark matter scale as a^{-3}, the radiation as a^{-4}, and the cosmological constant does not scale.

(a) Note that $a^{-1} \propto z+1$. Write a simple formula for the total density as a function of the red-shift z.

(b) Find the red-shift when the matter ($\rho_m = \rho_b + \rho_d$) matched the cosmological constant, and when the cosmological constant was only 1% of the matter.

(c) Recombination (to be studied soon) occurred at $z+1=1092$. Find the ratio of matter to radiation at this time. Is the cosmological constant important at this time?

(d) Primordial nucleosynthesis occurred around $z = 3.4\times10^8$. Find the density of just the ordinary matter at this red-shift. Compare to the density of air at standard temperature and pressure.

2. The current age of the universe was found in class, assuming radiation is irrelevant, and that $\Omega_m + \Omega_\Lambda = 1$, was given by

$$t_0 = H_0^{-1}\int_0^1 \frac{dx}{\sqrt{\Omega_m/x + \Omega_\Lambda x^2}}$$

(a) Given that x is simply $x = a/a_0$, argue that x corresponds to a simple function of the red-shift z. What must change about the upper limit of integration if we want to know the age of the universe t at red-shift z? (only the range of integration changes)

(b) Argue that if z is large, the term $\Omega_\Lambda x^2$ is irrelevant. Based on this, find a relationship for the age of the universe t as a function of red-shift z.

Graduate Problem: Only do this problem if you are in PHY 610

3. The age at any stage for the universe, assuming it is composed exclusively of matter and dark matter, is given by an integral of the form

$$t_0 = H_0^{-1} \int \frac{dx}{\sqrt{\Omega_m / x + \Omega_\Lambda x^2}}$$

   (a) What would be the limits if we want to know how long it takes from now until the universe is infinite in size? Convince yourself that this will take infinite time.

   (b) The formula above assumes that the dark energy does not scale, that is, that $\rho_\Lambda \propto a^0$. Suppose instead that $\rho_\Lambda \propto a^n$, with n a small positive number. How would the integral change?

   (c) Since dark energy currently dominates, and will do so more in the future, ignore Ω_m / x. Perform the integral you found in part (b), assuming n is positive.

   (d) We currently have $\Omega_\Lambda = 0.6889$ and $H_0^{-1} = 14.4\ \text{Gyr}$. Observation suggests $n = 0.08 \pm 0.09$. Assuming $n < 0.10$, what is the minimum time until the end of the universe? This end is called the "Big Rip."