ECN 215 – Expecation and Variance

The expected value of a discrete random variable X with possible values x_i, $i = 1, 2, \ldots, n$ is

$$E(X) = \sum_{i=1}^{n} P(X = x_i) \times x_i \tag{1}$$

That is, it's the probability-weighted sum of the possible values.

The variance of X is, using the same notation,

$$\text{Var}(X) = \sum_{i=1}^{n} P(X = x_i) \times (x_i - E(X))^2 \tag{2}$$

That is, the probability-weighted sum of the squared deviations of X from its expectation, $E(X)$.

Now, as we see from (1), the expectation of something is just the probability weighted sum of that something, so (2) can equally well be written as

$$\text{Var}(X) = E\left[(X - E(X))^2\right] \tag{3}$$

Expanding the square gives

$$\begin{aligned}\text{Var}(X) &= E\left[X^2 - 2XE(X) + E(X)^2\right] \\ &= E(X^2) - 2E(X)^2 + E(X)^2 \\ &= E(X^2) - E(X)^2\end{aligned}$$

Note that $E(X^2)$ and $E(X)^2$ are *not* the same thing. For example, when rolling a fair die.

X	X^2
1	1
2	4
3	9
4	16
5	25
6	36
$E(X) = 3.5$	$E(X^2) = 15.1667$

So $E(X)^2 = 3.5^2 = 12.25$, and $\text{Var}(X)$ is then $15.1667 - 12.25 = 2.917$.