The Solow Growth Model

The (Robert) Solow growth model presents a highly simplified account of economic growth. Nonetheless, it does give us some grip on the factors governing long-run macroeconomic outcomes, in particular the dynamics of the accumulation of capital.

1 Production function

We begin with a Cobb–Douglas production function

$$Y = AK^{\alpha}N^{1-\alpha} \tag{1}$$

where A is a "state of technology" multiplier, K is the stock of capital, and N is the labor input (number of workers). The exponent α is a positive fraction ($0 < \alpha < 1$). The fact that the exponents on both capital and labor are fractional means that we have diminishing marginal returns to each factor, if it is varied while holding the other one constant. The fact that the exponents sum to 1 means that we have constant returns to scale. That is, if we increase both N and K by x percent, output will also increase by x percent.

For long-run analysis we generally want to work in per-capita terms, so let's divide the production function by N:

$$\frac{Y}{N} = A\frac{K^{\alpha}N^{1-\alpha}}{N}$$

After a little algebraic manipulation this can be written as

$$\frac{Y}{N} = A\left(\frac{K}{N}\right)^{\alpha}$$

Using the shorthand $y \equiv Y/N$ (output per worker) and $k \equiv K/N$ (capital per worker), we get

$$y = Ak^{\alpha} \tag{2}$$

2 The steady state

Now let's consider what is needed to hold capital per worker constant over time.

First, suppose some fraction δ of the capital stock wears out or becomes obsolete each year. We call δ the depreciation rate. It is likely to be a fairly small figure, say around 0.03. An annual amount of investment δK is then needed just to keep K steady. (This equals an annual investment of δk in per-worker terms.)

Second, suppose that population and labor force are growing at an annual rate n. Then to keep capital *per worker* steady we'll need additional investment of nk (so that the people newly entering the workforce have just as much capital to work with as the existing workers).

It follows that we need annual investment of $(n+\delta)k$ to keep k steady. Any investment in excess of this value will raise the amount of capital per worker, while if actual investment falls short of this value then k will shrink over time.

How much investment goes on? We'll suppose that aggregate saving amounts to a fraction s of aggregate output; and in the first instance we'll assume that all of this saving goes into investment. Note also that in the context of the Solow model we abstract from the "Keynesian" problem of Aggregate Demand: we assume that the economy is always at full employment (or real GDP equals potential).

The points we have mentioned so far are illustrated in Figure 1. The top line shows output per worker (y) as a function of capital per worker (k) for a given state of technology. The sy line shows investment as a constant

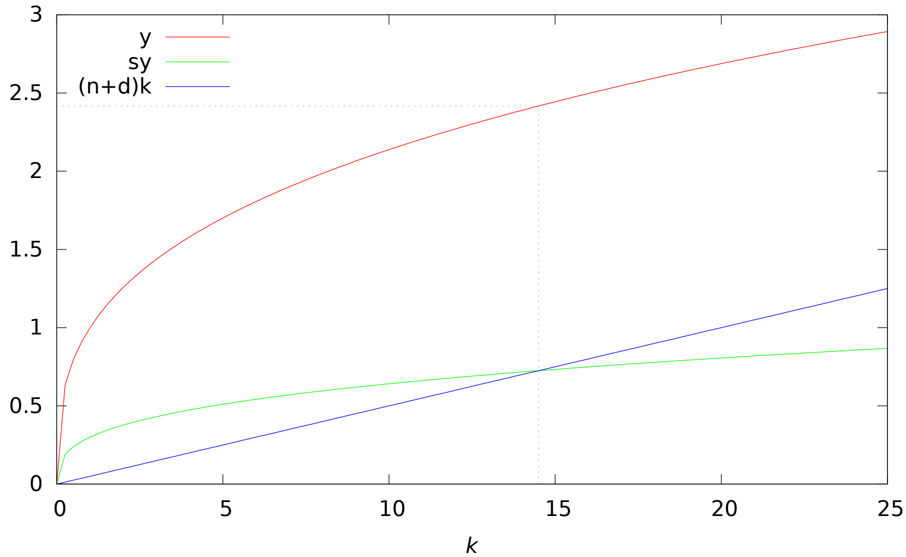

Figure 1: The steady state

fraction of y. And the $(n+\delta)k$ line shows the level of investment required to hold k at any given value. The figure uses the numerical values $\alpha = 0.33$, $\delta = 0.03$, $n = 0.02$, $s = 0.3$ and $A = 1$.

Here the steady state is at $k^* = 14.5$ and $y^* = 2.4$. At all points to the left of $k = 14.5$, actual investment is greater than $(n+\delta)k$, so the economy will "migrate" to the right: both capital per worker and output per worker will increase until the steady state is reached. Note that this will not be a quick process. Capital accumulates gradually, and if an economy starts out substantially below its steady state it may take decades to reach the steady state.

3 Analysis

In the context of the Solow model it is generally assumed that the state of technology (as represented by the coefficient A) is a world-wide phenomenon: all countries have access to current technology. It's also assumed that the depreciation rate, δ, is in common across countries. That leaves the saving rate, s, and population growth, n, as the factors that vary by country.

What's the effect of a higher rate of saving (and investment)? This shifts the sy line to a higher position; from the diagram we can see this means that it will intersect the line $(n+\delta)k$ further to the right. So if a country puts a larger fraction of its output into investment, other things equal, it will have higher steady-state values of capital per worker and output per worker. Note that in this model, while a high saving rate raises a country's steady-state k and y it does *not* give a country faster growth forever. (Once a country reaches its steady state, further growth of output per capita is limited to the rate of improvement in technology.)

And what's the effect of faster population growth? This gives the line $(n+\delta)k$ a steeper slope. It will therefore intersect the sy line further to the left. So if a country has fast population growth, other things equal, it will have *lower* steady-state values of capital per worker and output per worker.

These are the basic predictions of the Solow model. How well do they agree with the data? Statistical studies (such as the well-known article by Mankiw, Romer and Weil in the *Quarterly Journal of Economics*, 1992) suggest that model does quite well. Mankiw and his co-authors took real GDP per capita as the dependent variable and the key Solow variables—the saving rate and population growth—as the independent variables. In a cross-section of many countries, both advanced and less-developed, they found that the Solow variables accounted for about 60 percent of the variation in real GDP per capita. As the model predicts, they found a statistically significant positive coefficient on the saving rate, and a significant negative coefficient on the population growth rate.