The Multiplier Model

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March 3, 2021

1 Introduction

The basic idea behind the multiplier model is that—up to the limit set by "full employment" or potential GDP—the actual level of employment and output depends on the state of aggregate demand (AD). A shortfall in aggregate demand will push an economy into recession; an increase in AD can pull the economy back up to potential. (And an excessive level of AD is likely to cause inflation.)

The multiplier represents the ratio of the overall increment of real GDP to the increment of some independent expenditure variable,[1] for example investment (where the increment can be positive or negative, i.e. a decrement). Where the multiplier is greater than one, the "extra" change in aggregate expenditure is accounted for by an induced change in consumption. In all cases, the possibility of a *positive* multiplier effect rests on the assumption that there are unused resources—that is, the economy is operating short of full employment. If the economy is already at full employment, any increase in one category of real expenditure must be offset by a reduction in some other category or categories of real expenditure, or in other words the multiplier is zero.

Notation: the symbols used in the following discussion are defined thus:

Y	aggregate income or output
AD	aggregate demand
C	aggregate consumption
I	aggregate investment
G	government purchases
T	net tax revenue
X	total exports
Z	total imports

All of the above are assumed to be measured in real terms.

Planned and actual investment

Before proceeding we need to clarify something about investment spending, I. Total investment includes as one component "inventory investment," that is, the change in the stocks of goods waiting to be sold (or raw materials waiting to be processed) from one period to the next. One special feature of inventory investment is that it may sometimes be negative (meaning that stocks of unsold goods are falling). Another special feature is that it may be *unplanned* or *unintended*. For example, consumer spending falls unexpectedly and firms find that they have amassed a greater quantity of unsold goods, when they had no intention of doing so. We'll use this language: *actual* investment includes any unintended change in inventories, while *planned* investment excludes such changes. We'll use the special symbol I_p for planned investment.

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[1] "Independent" here means, not in any direct way a function of aggregate income.

2 The simplest case

We start with the simplest case, where we ignore (or set to zero) both the government sector and international trade. That leaves private-sector consumption and investment as the only categories of real expenditure, and we can write aggregate demand as

$$AD = C + I_p \tag{1}$$

where I_p represents *planned* or *intended* investment (excluding unintended changes in inventory). The consumption function is

$$C = a + bY \qquad 0 < b < 1 \tag{2}$$

and the short-run macroeconomic equilibrium condition is

$$AD = Y \tag{3}$$

That is, firms in the aggregate are producing what there's a demand for, neither more nor less.

If we substitute (2) into (1) and use (3) we obtain

$$Y = a + bY + I_p$$

which can be solved for Y as

$$Y = \frac{1}{1 - b}\ (a + I_p) \tag{4}$$

the "reduced form" of our simplest model. (I will give a step-by-step solution for the most complex case, in section 4 below. Presumably if you can follow that, you can fill in the steps for yourself in the simpler cases.) This model incorporates a multiplier of $1/(1 - b)$.

Numerical example: if the marginal propensity to consume, b, is 0.9 then the multiplier is

$$\frac{1}{1 - 0.9} = \frac{1}{0.1} = 10$$

and any change in investment will be multiplied tenfold in its effect on equilibrium GDP (within the full-employment limit as mentioned in section 1).

3 Introducing the government

We now introduce a government which carries out real expenditure G and collects real net tax revenue T. (Net tax revenue is tax revenue minus "transfers" such as social security and welfare payments; these transfers do not count as part of G since they are not payment for currently produced goods or services.)

We will represent net tax revenue as

$$T = tY \qquad 0 < t < 1 \tag{5}$$

This says that the government collects as tax some fraction t of the GDP.

Government expenditure must be added into aggregate demand, so we replace (1) with

$$AD = C + I_p + G \tag{6}$$

In addition, we must re-think the consumption function. With taxation in the picture, presumably consumption depends on post-tax or disposable income, $Y - T$. So we replace (2) with

$$C = a + b(Y - T) = a + b(Y - tY) = a + b(1 - t)Y \tag{7}$$

2

Using equation (7) in (6), we get

$$AD = a + b(1 - t)Y + I_p + G$$

and setting $AD = Y$ gives

$$Y = a + b(1 - t)Y + I_p + G$$

So

$$[1 - b(1 - t)]Y = a + I_p + G$$

and solving for Y gives

$$Y = \frac{1}{1 - b(1 - t)} \ (a + I_p + G) \tag{8}$$

which is the reduced form of our second model.

Numerical example: If $b = 0.9$ and $t = 0.3$ (a uniform 30 percent rate of income tax), we get

$$\frac{1}{1 - b(1 - t)} = \frac{1}{1 - 0.9(1 - 0.3)} = \frac{1}{1 - 0.9(0.7)} = \frac{1}{1 - 0.63} = \frac{1}{0.37} \approx 2.703$$

With the same marginal propensity to consume as in the earlier example, the positive tax rate produces a substantial reduction in the size of the multiplier effect. If fluctuations in investment are a significant source of instability in the GDP (as seems to be the case), the smaller multiplier is desirable. The overall fluctuations in GDP will be less than they would be if the tax rate were zero.

The tax rate, t, is a government policy variable, and it will influence the equilibrium GDP via its influence on the size of the multiplier. For instance, a cut in t will increase the size of the multiplier, and will therefore raise equilibrium GDP so long as I_p and G remain unchanged (and so long as we're not already at full employment).

4   Foreign trade

Our last multiplier model brings back the other important factor initially banished from discussion for the sake of simplicity, namely foreign trade. The full, final form of the aggregate demand equation is

$$AD = C + I_p + G + (X - Z) \tag{9}$$

where X denotes exports and Z denotes imports, so $(X - Z)$ is net exports. The rationale for this is that foreigners' purchases of our exports constitute an additional source of demand for our goods, while the import component of consumption and investment must be subtracted if we're trying to measure expenditure on domestically produced goods and services.

How should we treat X and Z in the context of our macro model, given that we're trying to strike a balance between simplicity and realism? We'll take exports to be an independent variable, on the grounds that they depend primarily on the level of demand in *other countries*. Imports, on the other hand, depend in an important way on the level of income and expenditure in the home economy: if we're spending more overall, we're likely to be spending more on imports too. The simplest way to capture this is to make imports proportional to GDP:

$$Z = zY \qquad 0 < z < 1 \tag{10}$$

Using (10) in (9) we get

$$AD = C + I_p + G + X - zY \tag{11}$$

From this point we proceed as before. First set the right-hand side of (11) equal to Y, as per equation (3).

$$Y = C + I_p + G + X - zY$$

Then, using the consumption equation (7), we get

$$
\begin{aligned}
Y &= a + b(1-t)Y + I_p + G + X - zY \\
Y - b(1-t)Y + zY &= a + I_p + G + X \\
[1 - b(1-t) + z]\, Y &= a + I_p + G + X
\end{aligned}
$$

So, finally, the reduced form is

$$
Y = \frac{1}{1 - b(1-t) + z}\, (a + I_p + G + X) \tag{12}
$$

Numerical example: With $b = 0.9$ and $t = 0.3$, as in the previous example, now let $z = 0.14$. That is, suppose 14 percent of expenditure goes on imports. In that case the multiplier is

$$
\frac{1}{1 - b(1-t) + z} = \frac{1}{1 - 0.9(1 - .3) + 0.14} = \frac{1}{1.14 - 0.63} = \frac{1}{0.51} \approx 1.96
$$

which is a roughly realistic value for the US economy. This multiplier is smaller again than the one after we first introduced the positive tax rate. Well, actually, the *global* multiplier is not any smaller, but from any one country's point of view the multiplier effect is reduced, in the sense that part of it "leaks" abroad, stimulating production in other countries (by augmenting their exports, which are our imports).

5   Some implications

The multiplier model, particularly in its most developed form, offers many insights into the working of the macroeconomy, some of them at variance with opinions frequently expressed by financial journalists and politicians (who should know better!). Here is a sampling. Note that you should not think of the following as independent "points to memorize": they follow directly from a correct understanding of the models set out above (plus a little general knowledge).

- The government budget balance, $G - T$, depends not only on *fiscal policy* (that is, the setting of government spending, G, and the tax rate, t) but also on the level of income. Other things equal, a rise in GDP will, by raising net tax revenue ($T = tY$), reduce the budget deficit or augment the budget surplus, while a fall in GDP will reduce the budget surplus or augment the budget deficit. If an increase in the budget deficit is brought on by recession, and the government reacts by cutting government spending or raising taxation (in an effort to eliminate the deficit), the effect will be a worsening of the recession. This sort of policy worsened and prolonged the Great Depression of the 1930s, and there is a danger that it will do so again today.

- The level of net exports depends jointly on (a) the level of exports, X, (b) the propensity to import, z, and the level of home-country GDP. Given X and z, a rise in GDP will reduce net exports by raising $Z = zY$, hence "worsening" the trade balance (moving it towards deficit). A fall in GDP will "improve" the trade balance (raise net exports, by reducing imports). Therefore the trade balance should not be taken as a straightforward indicator of the "health" of an economy. An "impressive" trade surplus may be the result of a depressed home economy.

- A stronger currency (rise in value of the domestic currency in relation to other countries' currencies) will tend to restrict exports (by making them more expensive in foreign currencies) and promote imports (by making them cheaper in the domestic currency). But a fall in X will tend to depress GDP, while a rise in z will reduce the value of the multiplier, again depressing GDP. Therefore, although people often talk as if a strong currency were always a desirable sign of economic health, there is such a thing as a "too strong" currency.

- Individual countries can boost their GDPs by carrying out policies that promote exports and restrict the propensity to import (e.g. tariffs, import quotas, export subsidies). But the sum of net exports at the world level has to be zero (think about it). So one country's gain in this game is another's loss. If a government wants to stimulate home-country GDP, because it currently falls short of the full-employment level, they are better off increasing their own spending, or cutting taxes, or trying to find ways to boost private-sector investment spending (e.g. lower interest rates, assuming that interest rates have not already bottomed out).